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DESCRIPTIVE GEOMETRY

A TREATISE FROM A MATHEMATICAL STANDPOINT

TOGETHER WITH

A COLLECTION OF EXERCISES AND
PRACTICAL APPLICATIONS

BY

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PREFACE.

Descriptive geometry is essentially a mathematical subject. The application of its principles to the making of working drawings, however, and the modifications which are made to suit the contingencies of practice, have had a tendency to obscure this fact, and like other theoretical subjects it has suffered mutilation in the interest of short cuts to immediate practical uses. But does not technical education, after all, consist chiefly in an equipment of sound theory? It has been the author's purpose to refrain from any attempt to hold the student's interest by clothing a few principles with some immediate pratical application, but instead, to present a sound theoretical treatment. How well he has succeeded he leaves others to judge.

The principles are herein formulated under theorems, as in plane and solid geometry; illustrative problems are solved in accordance with these theorems and special constructions discussed. The plan of, at least, one well know text is followed of dividing all problems into two parts; the first of which is a statement of the geometrical principles and the theoretical solution called an analysis; the second is a description of the graphic solution, accompanied by a drawing. An important feature is added, however, of giving the statement of the geometrical conditions and the solution in the analysis in a general form, instead of being made to refer to a certain kind of problem exclusively.

As an illustration of the generalized treatment throughout, attention may be called to the discussion of the cone. A common conception of a cone is that of a right circular cone, or cone of revolution. A generalized definition is given, however, to include all the surfaces which may be generated by a right line moving so as to pass through a fixed point and touch a curve. Further, it is stated that every cone is generically a right cone, and may be specifically named from the shape of a cross section so taken that a perpendicular, let fall from the fixed point or apex to the plane of the section, will pierce the latter in the center, focus, or other characteristic point, as a cusp, point of inflection, etc., etc.

While the third angle is undoubtedly to be preferred for working drawings, it is not thought that descriptive geometry, as mathematics, has any concern with a particular angle. The illustrative problems used deal indifferently with all so that the emphasis can be laid upon sound theory.

Exercises for students are grouped in the back of the book and suitably designated as belonging to a certain part of the text. An appendix deals with the subject of approximate methods, which has no proper place in the body of the book.

The author wishes to acknowledge his indebtedness to the well known texts, notably those of Church and of McCord, and also in particular to Arthur G. Hall, professor of mathematics at the University of Michigan, for valuable service in examining the mathematical treatment of the text.

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DESCRIPTIVE GEOMETRY

CHAPTER I.

THE POINT.

1. Descriptive geometry is the science of representing forms, plane and solid, by projecting them upon two or more planes at right angles to each other with the aid of projecting lines perpendicular respectively to these planes, and it also consists of the solution of problems relating to the properties and magnitudes of the forms.

2. The planes upon which objects are projected are known as the *coordinate planes of projection.* They consist, in general, of vertical planes, and a horizontal plane. The vertical planes may be at right angles to each other and are then distinguished as the *front vertical* and the *end vertical planes* respectively.

Working drawings for guidance in construction are made according to the general principles of descriptive geometry. The projection upon the horizontal plane corresponds to the plan view or simply plan; the projection upon the front vertical plane corresponds to the front view, front elevation, or simply elevation; the projection upon the end vertical plane corresponds to the end view or end elevation, as these terms are variously used.

3. Lines and planes in descriptive geometry, are not assumed to be limited. A line, which may be designated by letters at two of its points, is nevertheless possessed of the property of direction which does not stop short of infinity. A plane, likewise, extends to infinity.

Hence parallel lines are said to have two points in common at infinity, and parallel planes to meet each other in a common line at infinity.

4. The parallel projecting lines spoken of in section 1 have a common point at infinity, hence it is said that the '*center of projection*' is at infinity. This point is also called the '*point of sight*'* because, were it possible for the eye to be in its position, the forms projected from it as a center would have their projections identical with themselves, point for point, line for line.

5. Perspective is a branch of descriptive geometry in which the 'center of projection' is at a finite distance from the plane of projection. It gives the kind of pictures, for example, that would be seen if what was beyond a window were traced upon it by a person standing on the opposite side. The eye is the 'center of projection,' the window the 'plane of projection.'

6. Descriptive geometry and perspective together are parts of that broader subject projective geometry, in which the properties of figures are studied by projecting them upon any plane or planes from any center of projection and where there is found to be a correlation of the

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*The 'center of projection' and 'point of sight' are both equally good designing terms used by different authorities.

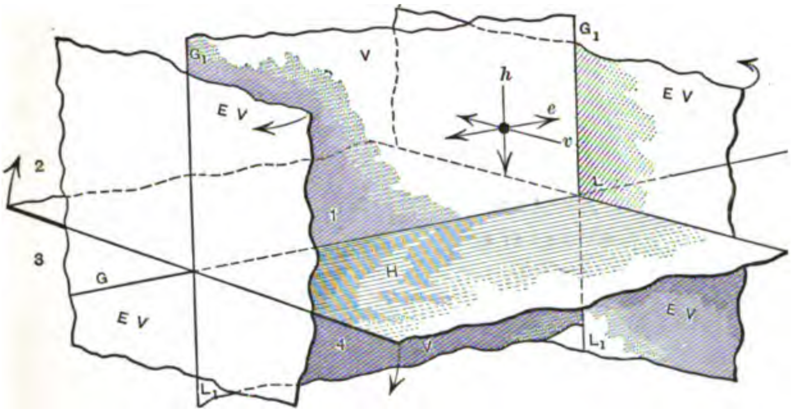

FIGURE 1.

properties of the different projections of a figure upon the different planes and from the different centers of projection.

7. **The two fundamental planes of projection** together with two end planes are shown in pictorial form or perspective in Figure 1. The front vertical and the horizontal planes [V and H planes] form together four dihedral angles. That which lies in front of V and above H is the 1st; that which lies back of V and above H is the 2nd, and so on, continuing in the same direction.

The line of intersection of these two planes is known generally as the ground line or abbreviated the G.L.

Objects are projected upon the front vertical plane by means of projecting lines such as shown at *v* which means that the center of projection for the object is at infinity in a direction in front of the vertical plane, i. e., opposite to that of the pointed end of the arrow. This center may be thought of as either above or below the H plane as the object is above or below it. It is in fact, at infinity in the horizontal plane, and in a direction perpendicular to the G. L.

Objects are projected upon the horizontal plane by means of vertical projecting lines such as at *h*, whence the center of projection is considered as infinitely distant above H in the V plane.

Objects are projected upon the end planes by means of horizontal projecting lines in the directions of the double headed arrow *e*, for the right hand plane [with forms in the first angle], from a center of projection which is at the left and for the left hand plane from a center of projection which is at the right, infinitely distant in the H plane.

8. **The V, H and end planes are unfolded** for the graphical representation of forms until they form one and the same plane, see Figure 2. Either V is revolved* about the G.L. as an axis until it coincides with H, or H is revolved about the G.L. until it coincides with V; the result in either case is the same. The curved arrows in Figure 1, connected with the H plane, show the direction of the revolution.

The end plane, which is at the right of any object,† is revolved about the ground line with the vertical plane or

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*An object is said to 'rotate' upon its own axis and to 'revolve' about any other axis not its own.

†In the theoretical treatment of the subject.

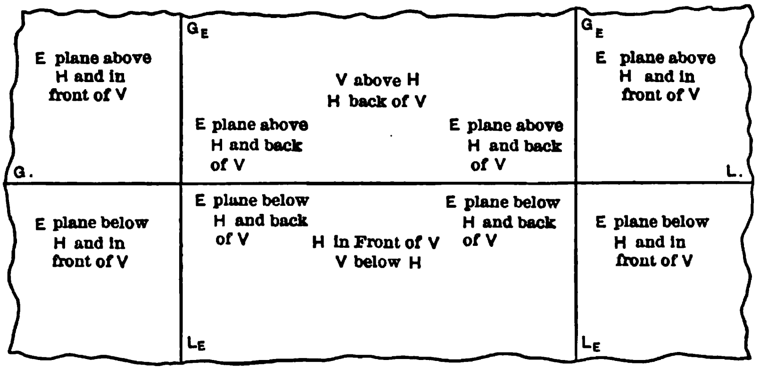
G$_E$
G$_E$
E plane above H and in front of V
V above H
H back of V
E plane above H and in front of V
E plane above H and back of V
E plane above H and back of V
G.
L.
E plane below H and back of V
E plane below H and back of V
E plane below H and in front of V
H in Front of V
V below H
E plane below H and in front of V
L$_E$
L$_E$

FIGURE 2.

$G_E L_E$ until it lies in V, that part which is in front of V going toward the right, that part which is back of V going toward the left. Similarly, the end plane, which is at the left of any object, is revolved into coincidence with V, the portion which is in front of V going towards the left.

9. **Let A be any point in space** in front of V and above H, i.e., in the first dihedral angle. Conceive a plane to be passed through it perpendicular to both H and V. It would correspond to an end vertical plane. This plane would cut V in a vertical line and H in a horizontal line, both perpendicular to the G.L. at the same point. When V and H are unfolded, these lines would become one and the same straight line perpendicular to the G.L.

The point A is projected on V by a projecting line perpendicular to V and lying in this end plane. It is a line parallel to H. A is projected on H by a projecting line perpendicular to H, also lying in this end plane. It is a line parallel to V. The piercing points of these projecting lines with the coordinate planes constitute the respective projections of A on those planes. From whence: *The two projections of a point are on a common perpendicular to the G.L.*

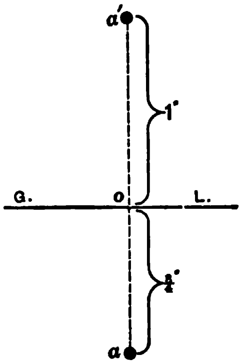

FIGURE 8.

When the V and H planes are unfolded, see Figure 3, *a* is the projection on H of the point A and *oa* is the projection on H of the projecting line of A on V. *oa* is equal to the distance of the point A from V. Also *a'* is the projection on V of the point A and *oa'* is the projection on V of the projecting line of A on H, and *oa'* is the distance of the point A from H. This is true for a point in either dihedral angle or when lying in either or both coordinate planes.

10. Descriptive geometry does not deal with forms themselves but with the projections of the forms.* Fig. 4 illustrates the projections of a point when variously placed with respect to the coordinate planes. A is in the 1st angle; B is in the 2nd; C is in the 3rd; D is in the 4th; E is in V above H; F is in V below H; G is in H in front of V; I is in H back of V; J is in the G. L. whence both projections coincide with the point itself. When the pro-

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* Hence the words 'the projection of' are unnecessary and when hereafter a form is spoken of, it means the projection of the form.

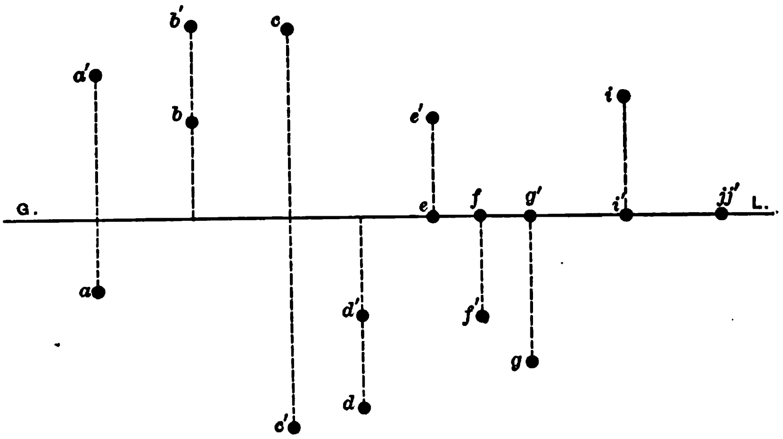

FIGURE 4.

jections and the forms happen to coincide, it avoids confusion to consider the projections only.

The positions shown in Figure 4 are known as the alphabet of the point, being the total of the various positions a point can have relative to the coordinate planes.

11 THEOREM I.

The two projections of a point fully determine the position of the point in space.

Proof:—If at the V projection of the point, a perpendicular is erected to the V plane, by hypothesis it will pass through the point in space. If at the H projection of the point, a perpendicular is erected to the H plane, it will also pass through the point in space. Hence the point itself must lie at the intersection of these two perpendiculars. But the perpendiculars are the two projecting lines of the point on the coordinate planes. Hence the point is fully determined.

If the point is in the H plane, the length of the perpendicular to the H plane is zero, and the point is its own projection. Likewise, if the point is in the V plane, it is its own V projection. If the point is in the G.L., both perpendiculars are zero and it is its own V and H projection.

12. Axes of reference may, for convenience in solving problems, be taken to coincide with the lines *oa* and *oa′* of Figure 3, the line *oa′*, as a portion of a y axis and *oa* as a portion of an x axis. Hence, the position of the point A can be specified numerically by coordinates. For example, referring to Figure 3, A = 1, $\frac{3}{4}$, meaning that it is 1 inch above H and $\frac{3}{4}$ inch in front of V, or its V projection is 1 inch above the G.L. and its H projection is $\frac{3}{4}$ of an inch in front of the G.L.

The G.L. may also be taken as a z axis, being perpendicular to the plane of the x and y axes at their intersection or origin, so that the exact location of points on a drawing can be specified by three coordinates, z, y and x, given in the order named; the z can be a distance from any convenient reference point, preferably one to the left as the border line of a sheet of drawings, the y as + or — according as the point is above or below H and x as + or — according as the point is in front of or behind V.

For example, in Figure 4, C can be specified as C = 1, — $1\frac{1}{16}$, — 1, where 1 means the z measured from the dotted line connecting a' and a; A can be specified as A = 0, $\frac{3}{4}$, $\frac{3}{8}$. The other points specified similarly, are: B = $\frac{1}{2}$, 1, — $\frac{1}{2}$; D = $1\frac{3}{8}$, — $\frac{1}{2}$, 1; E = $1\frac{3}{4}$, $\frac{1}{2}$, 0; F = 2, — $\frac{1}{2}$, 0; G = $2\frac{1}{4}$, 0, $\frac{3}{4}$; I = $2\frac{3}{4}$, 0, — $\frac{5}{8}$; J = $3\frac{1}{4}$, 0, 0.

When the *z* coordinate for any point is given as zero, let it be understood that the *xy* plane for the point may be taken anywhere, and the other points following are then measured from this plane as origin.

If either the *y* or the *x* coordinate of any point is missing and in its place is a question mark or a dash, it indicates that the coordinate omitted is to be found through other data in the problem.

Exercises.

1

Locate the following points, designating the projections by small letters, as *a′* for the V projection and *a* for the H projection, and state how each is related to the coordinate plane of projection: A = 0, 1, —½; B = ½, —½, —1; C = 1, 1, ½; D = 1½, —1, ½; E = 2, 1, 0; F = 2½, 0, —1; G = 3, 0, 1; I = 3½, 0, 0.

2

Locate the following points: A = 0, ⅝, ½; B = ¼, — ¾, — ½; C = ½, ¾, — ½; D = ½, — ¼, ⅝; F = ¾, 0, ¾; G = ¾, ⅜, 0; I = 1¼, 0, 0. Project the same upon an end vertical plane at the right whose $G_E L_E$ cuts the G.L. at the point O = 2¼, 0, 0.

Construction. Figure 5 shows the result. Since the end plane is perpendicular to V and H, the point A is projected upon it by means of a projecting line parallel to V and H. This projecting line will pierce the end plane in a point which is as far from V and H as the point A is distant from those planes, i. e., as far from the ground line in each case between the V, the H, and the end planes. When the end plane is folded into coincidence with the V plane, the projection of A upon it is a point at a distance *oa* from the

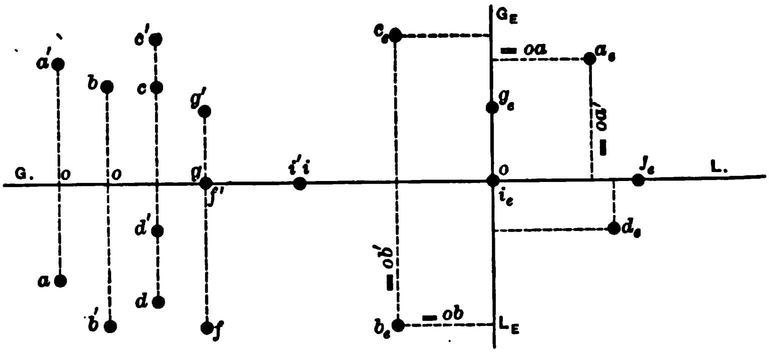

FIGURE 5.

G_E L_E and a distance *oa'* above the G.L. The G.L. is coincident with the revolved position of the ground line between the end plane and the H plane. That which is at the left of $G_E L_E$ is that portion of the end plane which lies beyond the V plane. So when considering the projections upon the end plane alone, G_E. L_E is the V plane in end projection and the G.L. is the H plane seen in end projection so formed by looking at the end plane toward the right in the direction of the G.L. of the two coordinate planes.

Likewise the end projection of the point B is found at a distance *ob'* below the G.L. and a distance *ob* to the left of the G_E L_E, being in the 3rd, dihedral angle, and so on for the other points.

3.

Draw the projections of the following points: A, 1'' below H and ⅞'' in front of V; B, in the 3rd dihedral angle 1½'' from both V and H; C, 1⅛'' behind V and ¾'' below H; D, in H, 1⅛'' behind V; E, in V, 1¾'' below H. Project these also upon an end plane whose G_E. L_E. may be chosen at pleasure.

CHAPTER II.

THE LINE AND PLANE.

13. A plane is projected upon the coordinate planes only by the projection of points or lines lying in it or by drawing its lines of intersection with the coordinate planes, designated generally as traces, and specifically the V and H traces.

Since a plane can intersect a line in only one point, the H and V traces of a plane will intersect upon the G.L.

If a plane is oblique to H and V and also to the G.L., its H and V traces are inclined to the G.L., meeting it in a finite point.

If a plane is parallel to the G.L. but oblique to H and V, its traces are parallel to the G.L. and intersect the latter at infinity.

If a plane is parallel to V its H trace is parallel to the G.L. and its V trace is at infinity. Similarly, if a plane is parallel to H, its V trace is parallel to the G.L. and its H trace is at infinity.

If a plane passes though the G.L. then both of its traces coincide with the G.L., and, referred only to its traces, the plane is indeterminate.

Figure 6 shows the several positions. 1 and 1a are

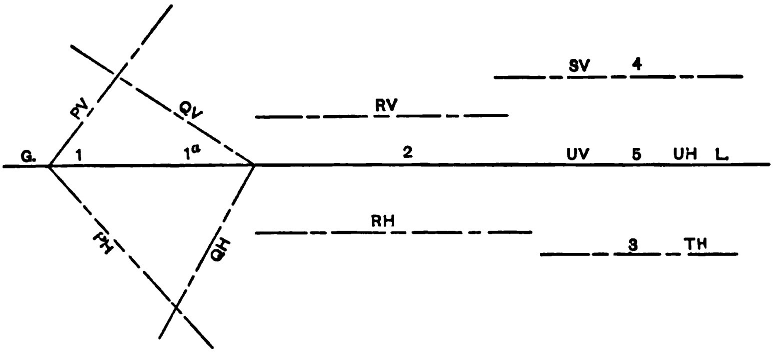

FIGURE 6.

the traces of a plane oblique to H and V and to the G.L. 2 is a plane parallel to the G.L. 3 is a plane parallel to V. 4 is a plane parallel to H. 5 is a plane passing through the G.L.

14. The alphabet of the plane is an expression used to designate the possible positions of a plane with respect to the coordinate planes. There are thirteen such positions.

(1). A plane oblique to the G.L. as in 1 or 1a of Figure 6.

(2). A plane parallel to the G.L. and cutting through the 1st, 2nd and 3rd dihedral angles.

(3). A plane parallel to the G.L. and cutting through the 1st, 4th and 3rd dihedral angles.

(4). A plane parallel to the G.L. and cutting through the 2nd, 1st and 4th dihedral angles.

(5). A plane parallel to the G.L. and cutting through the 2nd, 3rd and 4th dihedral angles.

(6). A plane perpendicular to H and oblique to V.

(7). A plane perpendicular to V and oblique to H.

(8). A plane perpendicular to both H and V.

(9). A plane passing through the G.L.

(10). A plane parallel to V and passing through the 1st and 4th dihedral angles.

(11). A plane parallel to V and passing through the 2nd and 3rd dihedral angles.

(12). A plane parallel to H and passing through the 1st and 2nd dihedral angles.

(13). A plane parallel to H and passing through the 3rd and 4th dihedral angles.

15. The following notation will be followed throughout the text and in the exercises at the end of the book.

A point in space is designated by a capital letter, as A, its projections by a small letter; if the V projection a' and the H projection simply a; if the end projection a_e.

A line is specified in general by two points on the line.

A plane is designated by lettering its traces with a capital letter followed by V, H or E according as it is the vertical trace, horizontal trace, or a trace with an end plane; the last letters of the alphabet will, in general, be used for this purpose. The letter does not specify any point on the trace.

The line of intersection of the V and H planes is called the G.L. The line of intersection of V or H and an end plane is called the $G_E.L_E$.

16. The following line conventions are recommended: The two projections of a point are connected by a dotted line, the dots to be about $\frac{1}{16}$ inch long, with $\frac{1}{32}$ inch spaces between the dots.

The G.L. and given lines, *light*, *solid.*

The Required lines, *heavy*, *solid.*

The projecting lines, construction and hidden edges of solids, *dotted lines.*

Auxiliary lines, *dash* with dashes about $\frac{1}{4}$ inch long and $\frac{1}{16}$ inch spaces between dashes.

Traces of given planes, *dash and dot;* $\frac{1}{4}$ inch dashes with $\frac{1}{16}$ inch dots and $\frac{1}{32}$ inch spaces.

Traces of required planes, *the same*, *heavy.*

Axes of revolution and axes of solids, *dash and two dots.*

It will be found helpful in the earlier part of the work to show all parts of forms in the 1st angle as solid lines and those in the other angles dotted.

17. THEOREM II.

If through any line in space a plane is passed perpendicular to a coordinate plane, it will intersect that coordinate plane in a line which is the projection of the given line upon the coordinate plane.

> **Proof:**—For this projecting plane, by definition of projection, contains the projecting perpendiculars of all points of the line upon the coordinate plane, the locus of their intersections with the coordinate plane constitutes the trace of the plane upon that coordinate plane.

18. Corollary 1. A line is, in general, fully determined when its projections are given, because if, through one of the projections of the line, a plane is passed perpendicular to that coordinate plane, it will contain the line, and if through the corresponding* projection a plane is passed perpendicular to the corresponding coordinate plane, it will also contain the line. Hence the line itself must be identical with that line which is common to both planes, namely their intersection.

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* Note the use of the word 'corresponding' occurring throughout the book; it facilitates the generalized treatment, makes processes applicable to either coordinate plane.

19. Corollary 2. If the projections of two points upon one of the coordinate planes are connected by a line, it is that projection of the line joining the two points in space. Similarly, if the corresponding projections of the points are connected by a line, it is the corresponding projection of the line connecting the two points in space.

A line connecting the projections of two points on a coordinate plane is indeterminate if it lies in an end plane for the two projecting planes of the line coincide.

20. To designate a point on any line, assume either projection of the point upon the same projection of the line, the other or corresponding projection is found upon the corresponding projection of the line, at the intersection with it, of a perpendicular to the G.L. through the assumed projection of the point.

21. The projecting planes of a line are useful in the solution of problems. Since the coordinate planes are perpendicular to each other and the projecting plane of a line is perpendicular to one of them, it follows that its corresponding trace is a line perpendicular to the G.L.

Figure 7 shows the projections of a line AB. SH and SV are the traces of its H projecting plane; TH and TV are the traces of its V projecting plane. That part of SV which is above the G.L. is the intersection of S with V

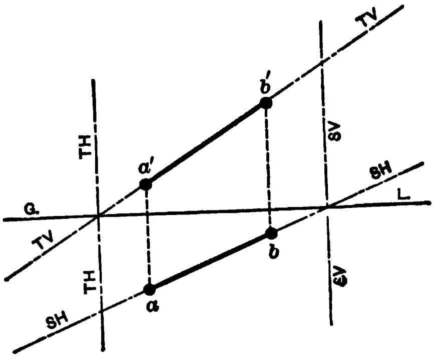

FIGURE 7.

above H and that part which is below the G.L. is the intersection of S with V below H. Similarly, that part of the H trace which is in front of the G.L. is the intersection of S with H in front of V* and that part of the H trace which is back of the G.L., is the intersection of S with H back of V.

The traces of a plane are not limited by the G.L.

22. THEOREM III.

The projections of the point of intersection of two lines are the points of intersection respectively of the projections of the two lines.

Proof:—The point of intersection of two lines, being common to both, can have but one projection on V and one projection on H. And since the point lies on both lines it must lie on the projection of both lines; the only point which satisfies these conditions is the point of intersection of the projection of the lines.

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* It is well to speak of the H projection of a point as in front of the G.L., not 'below,' also back of and not 'above.'

23. The form of proof to theorems, in descriptive geometry, differs in some respects from that used in mathematical subjects heretofore studied by the student, and a failure to readily grasp the significance or meaning of the various steps is a common source of difficulty. As in other mathematics, each step in descriptive geometry is capable of demonstration and proof, and should be so treated in all study. The proofs of steps should be followed mentally until the processes involved are performed unconsciously. A faithful adherance to this direction will remove much of the difficulties in the study of the subject.

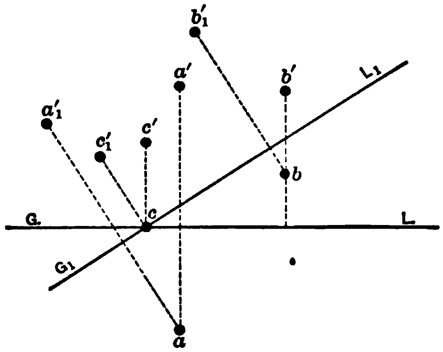

FIGURE 8.

24. The change of a coordinate plane is many times useful. Consider a change of the V coordinate plane.

Let A, B and C be three points shown in projection in Fig. 8. Project the same upon a new V plane or V_1, cutting H in the line $G_1.L_1$. The V_1 plane is revolved about the $G_1.L_1$. into coincidence with H, that which is above H going downward toward the left. The two projections of a point are on a common perpendicular to the G.L., hence A *is* projected on a perpendicular to the $G_1.L_1.$ through *a*. The distance of A above H remains the same, hence a_1' is a distance from $G_1.L_1.$ that a' is above the G.L. The point B is in the 2nd angle, but $G_1.L_1.$ cut H back of *b*, therefore, when b_1' is located on a perpendicular to the $G_1.L_1.$ at a distance above it that b' is above the G.L., it brings the point B in the 1st dihedral angle with respect to V_1. C lies in V and its H projection is at the intersection of the $G_1.L_1.$ with the G.L., hence, it also lies in the new V_1 plane at c' a distance from *c* that c' if from *c*.

The new H projections do not change; the distances of points from H are constant, hence the distances of their new projections in front of or back of the new $G_1.L_1.$ are constant.

25. A new coordinate plane, or H_1 plane, may be used with respect to either V or V_1. This is not a horizontal plane but is called an H_1 plane for convenience and to associate it with the V or V_1 planes. Let the ground line of the H_1 and V or V_1 planes be called $G_2.L_2.$ The H_1 projection of a point is found upon a perpendicular to the $G_2.L_2.$ through the V projection and at a distance from $G_2.L_2.$ that the H projection is from the G. L. Or the H_1 projection is found upon a perpendicular to the $G_2.L_2.$ through the V_1 projection and at a distance from $G_2.L_2.$ that the H projection of the point is from the $G_1.L_1.$

In this change the V or the V_1 projection does not change, nor the distance of the point from the V or V_1 plane. The distance of a point from the H_1 plane may be different from it distance from H.

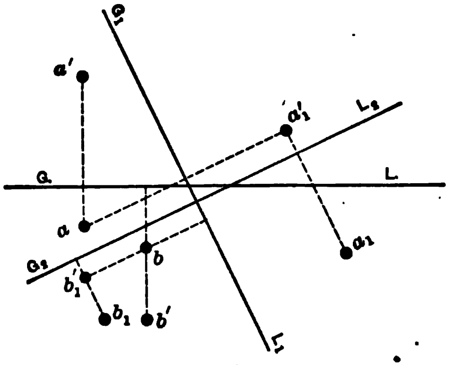

FIGURE 9.

Let A and B be two points in projection in Figure 9. Project them also upon a V_1 plane whose $G_1.L_1$. cuts the G.L. as shown. A new H_1 plane also cuts V_1 in a line G_2. L_2. at right angles to the G_1. L_1. a_1' is found at a distance above the $G_1.L_1$ that a' is above the G. L. and a_1 is found on a perpendicular to the $G_2.L_2$. through a_1' and at a distance from it that a is from the $G_1.L_1$. Similarly b'_1 is a distance in front of the G_1 L_1 , i. e., to the left, that B is from the H plane shown at b', and b_1 is a distance in front of G_2 L_2 that b is from the G_1 L_1 .

26. If a line is parallel to either coordinate plane, the trace of the projecting plane of the line on the other or corresponding plane of projection is a line parallel to the G.L.

If a line is parallel to both coordinate planes of projection then the traces of both projecting planes of the line are parallel to the G.L. Hence, when a line is parallel to

V its H projection is parallel to the G.L. and when it is parallel to H its V projection is parallel to the G.L. If it is parallel to both H and V it is parallel to the G.L. and both projections are parallel to G.L.

If a line is parallel to a plane all points of it are equally distant from that plane, hence the projection of a line on a coordinate plane to which it is parallel will show the true length of the line, and, by its angle with the G.L., the angle the line makes with the corresponding plane of projection.

If a line is perpendicular to a coordinate plane its projection on that plane is a point and its projection on the corresponding coordinate plane is a line perpendicular to the G.L. and equal in length to the line itself.

If a line is parallel to an end plane, it is necessary to project the line, by two of its points upon the end plane, to determine its length, direction, and its angles with either coordinate plane.

27. The trace of a line is the point of intersection of the line with any plane. Commonly, in descriptive geometry, it means the point of intersection of a line with one of the coordinate planes of projection.

Referring to Fig. 10, every point in the vertical projection of A B, being the vertical projection of some point in the line, shows that the point D in which this projection cuts the G. L. is the vertical projection of the point of the line which is in H*, that is, it is the vertical projection of

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* The G. L. contains the vertical projections of all points lying in the H plane, in fact it is the vertical projection of the H plane. It also contains the H projections of all points in the V plane, and it is likewise the H projection of the V plane.

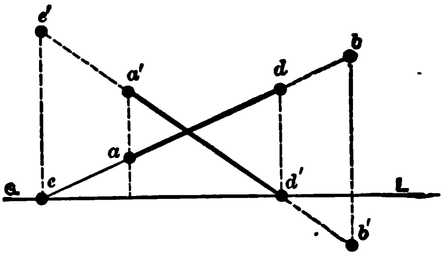


FIGURE 10.

the H trace of the line AB. The H projection of the H trace is on the H projection of the line. Similarly the point of intersection of the H projection of AB with the G. L., *e*, is the H projection of the V trace; the V projecttion of the V trace is on the V projection of the line, Hence may be derived the following:

28. To find the trace of a line with either coordinate plane, prolong the corresponding projection of the line until it cuts the G. L. It is the corresponding projection of the trace desired. The other projection of the trace will be found upon the other projection of the line.

The preceding is a general statement. It may be divided more specifically into a rule for finding the H trace of a line and one for finding the V trace of a line. The first of these can be stated as follows:

Prolong the V projection of the line until it intersects the G. L., which point is the V projection of the H trace; the H projection of the H trace is on the H projection of the line at the intersection of a perpendicular to the G. L. through the V projection.

Observe that neither of the two coordinate points alone is the trace sought, but the two together constitute the *projections of the trace.*

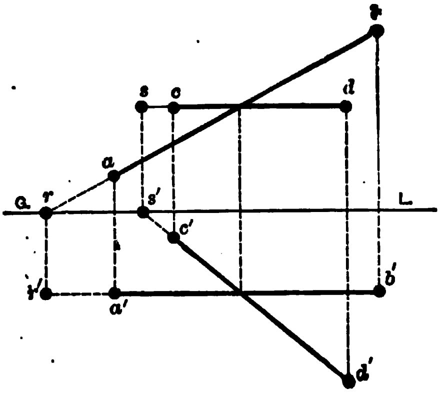


FIGURE 11.

29. Let AB and CD, in Fig. 11, be two intersecting lines the first of which is parallel to H and the second parallel to V. The point R is the V trace of the line AB. The H trace is at infinity. The V projection of the H trace will be on the G. L. at infinity and the H projection of the H trace will be on the H projection of the line at infinity. Similarly S is the H trace of the line CD. The H projection of the V trace is on the G.L. at infinity, the V projection of the V trace is on the V projection of the line at infinity.

Hence to make a general statement: When a line is parallel to either coordinate plane the projection of its trace on that plane is on that projection of the line at infinity; the corresponding projection of the trace is on the G. L. at infinity.

30. If a line is parallel to both coordinate planes its H and V traces are identical both upon the G. L. and the line at infinity.

If a line goes through the G. L. at a finite point, that point constitutes both projections of both traces.

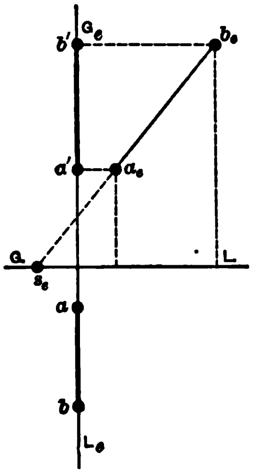


FIGURE 12.

If a line lies in an end plane its traces are in the ground line or G_E. L_E. of the end plane with the V or H plane. The rule is modified to suit the condition. For example see Fig. 12. AB is a line in an end plane. Revolve this plane about its G_E .L_E . into the V plane and get the end projection of AB at a_e b_e. To obtain the H trace of the line prolong the end projection of the line until it intersects the G. L. which is the revolved line of intersection of the end plane with the H plane. Let s_e be the end projection of the H trace. The H projection of the H trace will be on the H projection of the line at a distance above the G. L. that s_e is to the left or back of the G_E . L_E . The V projection of S will be on the G. L. at its intersection with the G_E . L_E . Similarly to obtain the V trace prolong the end projection of the line until it intersects the G_E L_E which is in V. The H projection of the V trace is in the G. L. at the intersection with G_E . L_E .

31. To find the traces of a line by means of an end plane, project the line upon the end vertical plane, then prolong this projection, if necessary, until it intersects the V plane in the $G_E.L_E.$, and the H plane in the G.L. The former is the V projection of the V trace, the latter is the revolved H projection of the H trace. The H projection of the V trace is in the G. L. and the H projection of the H trace is on the H projection of the line, a distance in front of or back of the G. L. that the point is in front of or back of V.

32. The proper study of descriptive geometry requires that every problem be divided into two parts, first *analysis*, second, *solution* or graphic demonstration. The first is the consideration of the geometrical principles involved, together with a description of the method of solution if it is a problem or of laying out the data if it is simply the graphic interpretation of the geometric conditions; its treatment should be general. The second concerns the graphical part and consists in solving the problem graphically according to the principles decided upon in the analysis or graphically interpreting the data if it be only the interpretation of descriptive geometry conditions.

The student is cautioned never to hasten to the second without doing full justice to the first. The analysis may involve the statement of descriptive geometry conditions, the geometric principles involved and a division of the main problem into parts each of which has its separate solution. However simple or complex the problem is, it should have its analysis preceding the graphical interpretation.

Verify geometrically all steps or processes by going over the most fundamental and elementary principles involved in each problem until the interpretation of the fundamentals no longer gives any trouble, but is dealt with as unconsciously as letters are formed to spell words in writing. The entire subject depends upon a few comparatively elementary concepts. These should be fully grasped before their varied application can be intelligently handled.

33. To draw a line containing a point in each of two given dihedral angles:—Locate the points in the specified angles and draw the projections of the line to contain the respective projections of the points.

A line can also be made to pass through any one of the dihedral angles severally, by a consideration of its traces and their proper location. For example, if a line goes from left to right, and from the 1st angle through the 2nd into the 3rd, its V trace must be above the G.L., and its H projection must touch the G.L. to the left of the V projection; its H trace must be back of the G.L., i.e., the perpendicular from the V projection of the H trace must meet the H projection of the line back of the G.L. If a line having the same direction goes from the 1st angle through the 4th into the 3rd, then the H trace must be in front of the G.L., i.e., the V projection must intersect the G.L. to the left of the H projection, and the perpendicular from the H projection of the V trace must intersect the V projection of the line below the G.L.

A line lying in four dihedral angles is a line lying in either coordinate plane and crossing the G.L. in a finite point.

34. The alphabet of the line is an expression used to designate the possible positions of a line with respect to the coordinate planes. The directions of a line are seven in number.

1. Inclined to H and V.
2. Parallel to H and inclined to V.
3. Parallel to V and inclined to H.
4. Parallel to H and V or parallel to the G.L.
5. Perpendicular or normal to V.
6. Perpendicular to H.
7. Inclined to H and V and perpendicular to the G.L.

With direction (1) a line may go (a) through the 1st, 4th and 3rd angles, (b) through the 1st, 2nd and 3rd angles, (c) through the 4th, 1st and 2nd angles, (d) through the 4th, 3rd and 2nd angles, (e) through the G.L. and from the 1st to the 3rd angles, or (f) through the G.L. from the 2nd to the 4th angles.

With direction (2) a line may go (a) through the 1st and 2nd angles, (b) through the 4th and 3rd angles, or (c) it may lie in H and hence cut the G.L.

With direction (3) a line may go (a) through the 1st and 4th angles, (b) through the 2nd and 3rd angles, or (c) it may lie in V and hence cut the G.L.

With direction (4) a line may lie (a) in the first angle, (b) in the 2nd angle, (c) in the 3rd angle, (d) in the 4th angle.

With direction (5) a line may go (a) through the 1st and 2nd angles. (b) through the 4th and 3rd angles, or (c) it may lie in H and go through the G.L.

With direction (6), a line may go (a) through the 1st and 4th angles, (b) through the 2nd and 3rd angles, or (c) it may lie in V and go through the G.L.

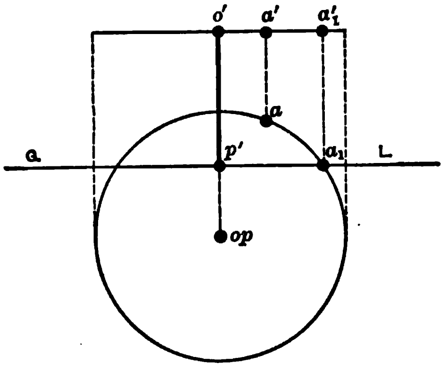


FIGURE 13.

With direction 7, a line may go (a) through the 1st, 4th and 3rd angles, (b) through the 1st, 2nd and 3rd angles, (c) through the 4th, 1st and 2nd angles, (d) through the 4th, 3rd and 2nd angles, (e) through the G.L., the 1st and 3rd angles, or (f) through the G.L., the 2nd to the 4th angles.

The student is advised to draw out these positions for himself as a drill in determining traces and as a reference chart for the earlier work in the subject

35. A point is said to revolve about a line as an axis when the path of the point is a circle whose plane is perpendicular to the axis and whose center is in the axis. A simple case is that of the revolution of a point about a line which is perpendicular to either coordinate plane. If the line or axis of revolution is perpendicular to H, the conditions are as shown in Fig. 13. A point A revolves about the line OP. The path of the point being a plane perpendicular to OP, which in turn is perpendicular to H, must be parallel to H, hence has its center at O and is a true circle in H projection. Its vertical projection is a

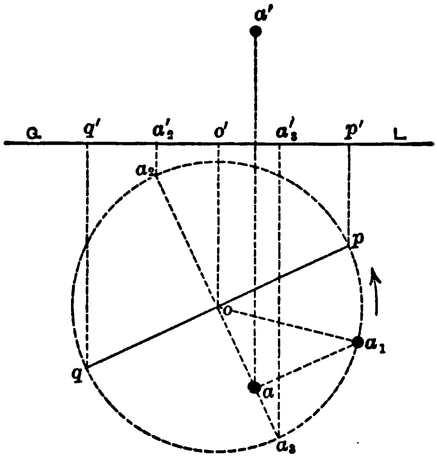


FIGURE 14.

limited straight line perpendicular to OP or parallel to the G. L. with center at o'. If the point A moves through any given angle, the angle in its true value is laid off with o as a center and the vertical projection of the point is found upon the vertical projection of the path. Let the point move toward V from the position A until it lies in the V plane. a_1 is its H projection, where the H projection of the path of the revolution crosses the G.L. and $a_1'$is the vertical projection upon the vertical projection of the path.

Another illustration is, when the point revolves about any line lying in either coordinate plane. Let A in Fig. 14 be such a point and QP the given line. The path of the point, being perpendicular to the axis, is projected on H in this illustration as a limited line perpendicular to $o\ p$ for OP lies in H and a plane perpendicular to H will be projected on H in a straight line. The center O of the path is horizontally projected where the H projection of the path cuts the H projection of the axis. The radius of revolution is the perpendicular distance of the point from the axis. This perpendicular is oblique, in this case, to both H and V. To state it in general terms *it is the hypotenuse of a right angled triangle of which one side is the perpendicular distance of the projection of the point on the plane of the axis from the axis and whose other side is the distance of the point from the plane of the axis.* In the example under consideration the first side mentioned is the distance oa, and the second side is the distance of a' from the G. L. This radius may be derived by auxiliary construction but it is better to get it in the following way: Conceive the plane of the path of the point to be revolved about its H projection into H that is about the line oa. A falls at a_1; $a\ a_1$ is the second side of the triangle just mentioned and the revolved position of the path is a circle with oa_1 as a radius and diameter $a_2\ a_3$. If A moves from the given position, in the direction of the arrow in revolved position, it falls in the H plane at a_2, a distance from o equal to $o\ a_1$. If it continue motion it will go through the 4th angle again falling into H at a_3.

Every point in the path of A shown in the figure as revolved into H is horizontally projected in the diameter $a_2\ a_3$ and vertically projected on perpendiculars through these points as shown in the figure. The lengths of the ordinates to the diameter $a_2\ a_3$ are the distances respectively that the points lie above or below H according to the direction of revolution of the path into coincidence with H. In this case all the points in the circle to the right of the diameter $a_2\ a_3$ stand for positions above H and those to the left for positions below H.

If it is desired to revolve the point A through a given angle about the line QP the angle can be laid off with o as a center and subtended by an arc of the circle $a_3\ a_1\ a_2$. The new position of the point would be horizontally projected by an ordinate to the axis and vertically by laying off the length of this ordinate above or below the G. L. as the case may be on a perpendicular to the G. L. through the H projection.

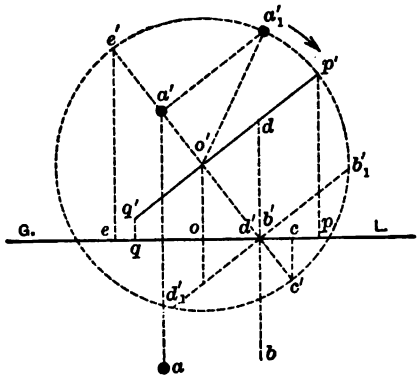


FIGURE 15.

Let A, Fig. 15, be a point in space to move about a line PQ in V, in the direction of the arrow shown in revolved position. The radius of revolution is the hypotenuse $o'\ a_1'$ and the revolved position of the path is shown at a'_1 b'_1 d'_1. A is in the 1st angle; if it moves to the position b_1' shown as revolved into V the V projection of the point is b' and the H projection at b is a distance below b' equal to the length of the ordinate $b_1'\ b'$. The point is in H in front of V. If the point moves from the position B to the position C it is in the 4th and 3rd angles. C is a point in V below H and hence c, in the G. L., is its H projection. If the point moves from the position C to the position D, it lies in the 3rd and 2nd angles. d_1' is the revolved position of the point lying in H back of V, *d′* is its V projection and *d* is a distance from the G. L. equal to the length of the ordinate $d'd_1'$. From D to E the point is in the 2nd angle. At E the point has come into V, hence its H projection is in the G. L.

The student is advised to copy the figure upon a somewhat larger scale and locate the projections of a few intermediate points in the revolution to familiarize himself with the principles involved.

36. To revolve a point about a line which is oblique to both coordinate planes is a much more difficult problem than the foregoing. If it should occur in any practical problem, a good solution is to change the coordinate planes; project the axis and point upon a V_1 plane parallel to the axis, or to project them upon a plane through the axis, and then proceed in the manner outlined.

37. A change of one coordinate plane for a line may be made as follows: A line may be projected upon any new V_1 plane (see figure 16) by finding the new V_1 projections of points of the line. If the $G_1.L_1.$ is taken perpendicular to the H projection of the line obviously the V_1 projections of all points of the line will be in a common perpendicular to the $G_1.L_1.$, and we have indeterminate projections of the line as we have when the line lies in an end plane.

Project the line upon a new vertical plane or V_2 (see Figure 16) to distinguish it from the one just mentioned, whose $G_2.L_2.$ is parallel to the H projection of AB. Obviously V_2 is parallel to the line since $G_2.L_2.$ is parallel to *ab* which is the H trace of the H projecting plane of AB. Finding the projection of AB on V_2 in $a_2^1\,b_2^1$, we have a

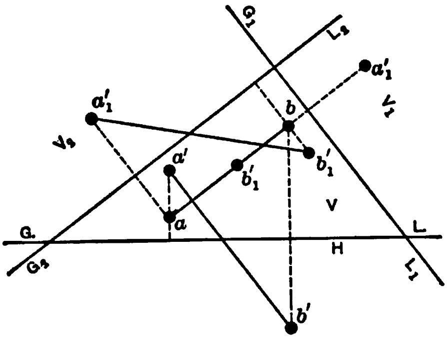


FIGURE 16.

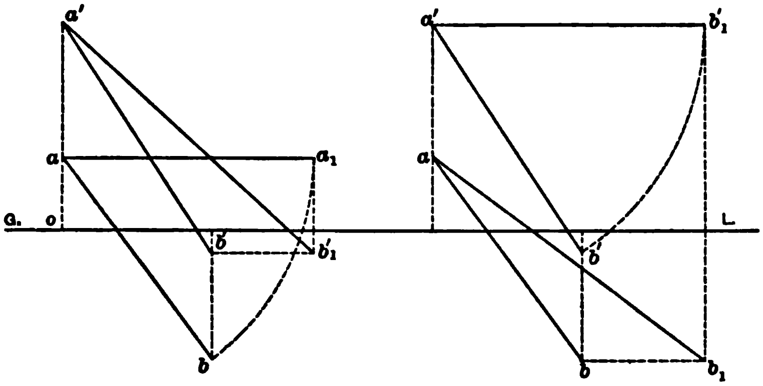
projection which is parallel to the line and of the same length as the line. This shows one of the uses of a change of coordinate plane.

To project a line upon a new V_1 plane whose $G_1 . L_1$. is parallel to the H projection of the line results projectively in the same way as considering the H projecting plane of the line. And since the V_1 projection is revolved about the $G_1 . L_1$. into coincidence with H, and further since a revolution about the V projection into V is similar to a revolution about the H projection into H, we may state as the analysis for finding the true length of a line by projection:

Analysis:—*Revolve the line about either projection into the coordinate plane of that projection. The revolved position is the true length required.*

38. To find the true length of a line by revolution.

Analysis:—*Revolve the line about the projecting line on V or H of any one of its points until the line is parallel to H or V. Its projection on the plane to which it is parallel, will give the true length required.*



FIGURES 17 AND 18.

Construction:—A convenient point is one extremity of the limited line. See Figure 17.

Revolve the line AB about the H projecting line *a'o'* of the point A on H[*] until *ab* is parallel to the G.L. The point *b* travels in the arc of a circle with *a* as a center. This circle being parallel to H, is projected on V as a straight line parallel to the G.L., whence b'_1 the intersection of such a parallel with a perpendicular to the G.L. through b_1 is the position of the point in vertical projection when the line is parallel to V. $a'b'_1$ is then the true length of the line.

Similarly, the line might be revolved about the V projecting line of either A or B until the line AB is parallel to H, whence, in its revolved position, the H projection would show the true length of the line. (See Figure 18.)

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[*] *a'o* is the vertical projection of the H projecting line.

CHAPTER III.

PROBLEMS IN POINT, LINE AND PLANE.

39. THEOREM IV.

If a line lies in a plane the traces of the line lie in the corresponding traces of the plane.

> **Proof:—By geometry, a line lies in a plane when every point of the line lies in the plane. The trace of a plane with a coordinate plane is a line lying wholly within the plane. It must intersect every other line of the plane, each in some point. Such a point being on another line of the plane, must be the trace of that line with the coordinate plane.**

The converse of this is not always true, for example, if a line goes through the G.L. at the point of intersection with the G.L. of any plane, the traces of the line will coincide with each other and lie upon the traces of the plane but the line will not necessarily lie in the plane unless some other point also does.

40. THEOREM V.

A given line is parallel to a plane when it is parallel to a line in the plane or when it lies in a parallel plane.

Proof:—Since from geometry parallels between parallels are equal, and conversely, if from any two points in the line parallels can be drawn which also terminate in the given plane and are of equal length, they will pierce the given plane in points which lie on a line parallel to the given line. Hence the given line is parallel to the plane. And since what is true of one line will also be true of either line, a line is parallel to a plane when it lies in a parallel plane.

41. Corollary. If a line is parallel to either coordinate plane, then any plane passed through the line will have its trace on that coordinate plane parallel to the projection of the line on that plane.

42. Every point in the V trace of a plane is a point in V as well as a point in the plane, hence the corresponding projection is in the G.L. Similarly every point in the H trace of a plane is a point in H as well as a point in the plane, hence the corresponding or V projection of this point is also in the G.L. To locate any other points lying in the plane conceive of a line or lines lying in it, passing through the point, and limited by the traces of the plane.

43. To pass a plane through three given points.

Analysis:—*Since the points lie in a plane they will lie upon lines of the plane passing through them. Hence if lines are drawn connecting the points the traces of two such lines are sufficient to locate the traces of the plane of the lines, that is the plane of the points.*

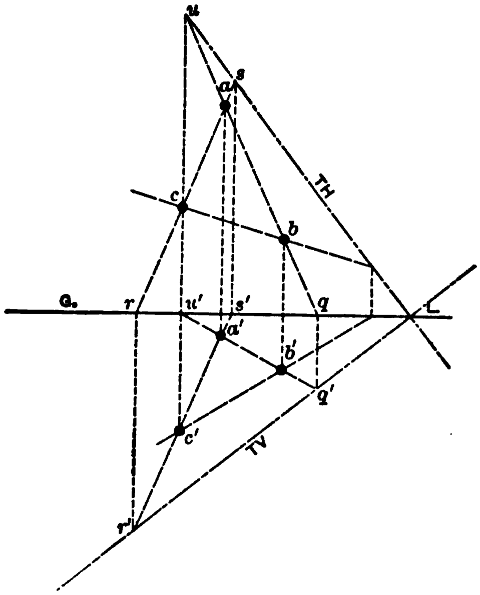

FIGURE 19.

Construction:—(See Fig. 19).

Let A, B and C be three points in projection. Connect A and B by a line, also A and C. Q is found to be the V trace of AB (by Sec. 28) and R the V trace of AC. The V trace of the plane of the three points passes through the points r' and q'. Similarly U is the H trace of the line AB and S is the H trace of the line AC, and the H trace of the plane of the three points goes through the points u s. If the work is done accurately the H and V traces will meet on the G.L. Hence, if one trace is obtained by the method described only one point is required in the other trace.

If one of the lines drawn connecting two of the three points is parallel to the G.L. then any plane passed through this line is parallel to the G.L. by Theorem V, Sec. 41, and the traces of the plane are parallel to the G.L. Hence, it is only necessary to locate one point in each trace.

If the traces of lines connecting three points in space do not come within available limits on the drawing, then any auxiliary lines may be taken to intersect those connecting the three points. These can be so chosen that their traces can be readily obtained.

If the lines connecting the projections of the points are perpendicular to the G.L., it is necessary to project the points upon an end plane; if one line connecting two points is perpendicular to the G.L., then an auxiliary line may be used.

The problem to pass a plane through a line and a point does not differ from the preceding. Draw an auxiliary line through the point to intersect the given line and proceed as before. A convenient auxiliary line to use is one which is parallel to the given line.

44. To draw the traces of a plane which shall contain a given line.

Analysis:—*Since an infinite number of planes can be passed through a line, draw, through the V and H traces of the line, the V and H traces of any plane which will intersect each other in the G.L.*

45. The horizontal and the vertical of a plane are respective lines in the plane, the first of which is parallel to H and the second is parallel to V.

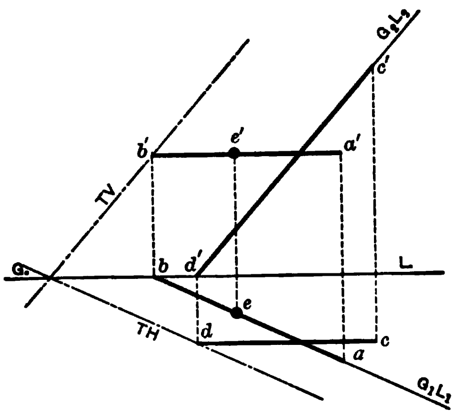

FIGURE 20.

For example, see Figure 20, the line AB is a horizontal of the plane T for its H projection is parallel to the H trace of the plane T, and its V projection is parallel to the G.L. And similarly, CD is a vertical of the plane T for its V projection is parallel to the V trace of the plane and its H projection is parallel to the G.L. These lines are useful in the solution of problems.

It is useful, sometimes, to consider a line as the intersection of two planes, for example, the line AB of Figure 20 is the line of intersection of the plane T with a V_1 plane which is parallel to the H trace of T and whose G_1. L_1. coincides with *ab*. Its vertical trace will be the line *bb'*. In like manner the line CD is the line of intersection of the plane T with a new H_1 plane which is parallel to the V trace of T and whose G_2. L_2. coincides with the line *c'd'*. Its H trace will be the line *d'd*.

46. To assume a point in a plane.

Analysis:—*Since lines lying in the plane and passing* *through the point have their traces in the respective traces of the plane, one projection of such a line may be assumed at will to contain one assumed projection of the point, and by means of the traces of the line, the corresponding projection may be found, which in turn contains the corresponding projection of the point.*

The horizontal or vertical of the plane are convenient lines to use for this purpose. E is a point in the plane T, Figure 20, found by the method described.

47. THEOREM VI.

The traces of parallel planes with a third plane are parallel.

> **Proof:** By hypothesis the traces each lie in the respective planes and also in a third plane together. By Theorm V, if through any two points of one of the lines or traces parallels can be drawn, lying in the same plane, and of the same length the line connecting their extremities will be parallel to the line or trace. But the two such lines which may be drawn to connect the points on the second trace are themselves in a third plane together with the first line, and the line connecting their extremities lies in the third plane and is parallel to the first line or trace.

48. Both coordinate planes may be changed with respect to a line. Let AB, Fig. 21, be the line to project upon any new V_1 plane and H_1 plane. Find the V_1 projection by Sec. 24. Let the new H_1 plane be chosen by its $G_2.L_2$, it is perpendicular to the V_1 plane. The points A and B are

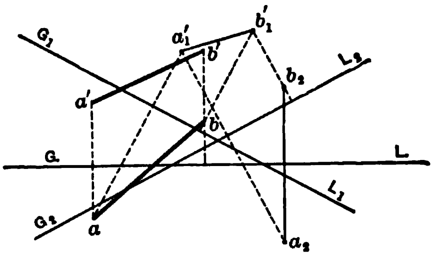

FIGURE 21.

projected upon it by drawing perpendiculars from their V_1 projections to the $G_2.L_2$. The distances of the points from V_1 are fixed and their projections upon any H_1 plane are as far from the ground line of that plane with V_1 as the H projections are from the ground line with V_1 hence a_2 and b_2 are on perpendiculars to $G_2.L_2$. through a'_1 and b'_1 and distant respectively from $G_2.L_2$. the amounts that a and b are from the $G_1.L_1$.

49. Change of one coordinate plane with respect to any oblique plane: Let T be any plane, see Fig. 22. Choose

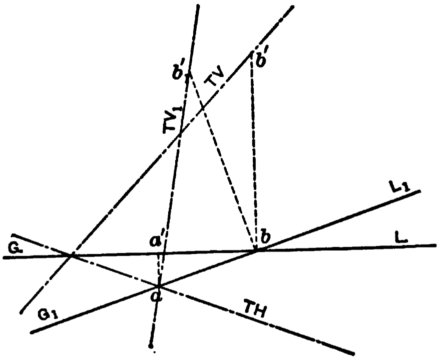

FIGURE 22.

any new V_1 plane with $G_1.L_1$. cutting TH in the point A. The plane V_1 cuts a line from V which is perpendicular to the G. L. because V_1 is perpendicular to H, this line is *bb'*. *b'* is also a point in T because it is in the V trace of T. The V_1 projection of the point B is on a perpendicular to the $G_1.L_1.$ through *b*, namly b'_1. The V_1 projection of A is *a* because the point lies at the intersection of TH and $G_1.L_1.$ It must moreover be a point in the new V_1 trace of the plane T. Hence ab'_1 is the V_1 trace of the plane T.

If $G_1.L_1.$ *had been taken perpendicular to TH,* by inspection, we can see that the plane T would have been perpendicular to V_1.

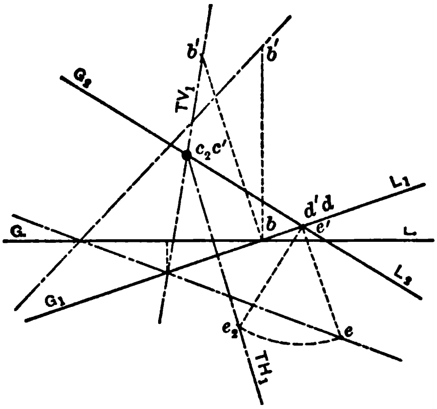

FIGURE 23.

50. Change of both coordinate planes with respect to a plane: Let T be any plane, Fig. 23. Find its V_1 trace as in the preceding paragraph. Now take any new H_1 plane cutting V_1 in the line $G_2.L_2.$ Where TV_1 cuts $G_2.L_2.$ is one point in the H_1 trace of the plane, i. e., C. The plane H_1 cuts a line from H which is perpendicular to V_1, because both planes are perpendicular to V; this line is DE. E is also a point in T because it is in the H trace of T. The H projection of E is on a perpendicular to $G_1.L_1.$

through e'. The H_1 projection of E is on a perpendicular to $G_2.L_2.$, a distance from it equal to ***d e.*** It is a point in the new H_1 trace of the plane T. C is another point in the H trace. Hence, the line TH_1 connecting c_2 and e_2 will be the H_1 trace of the required plane.

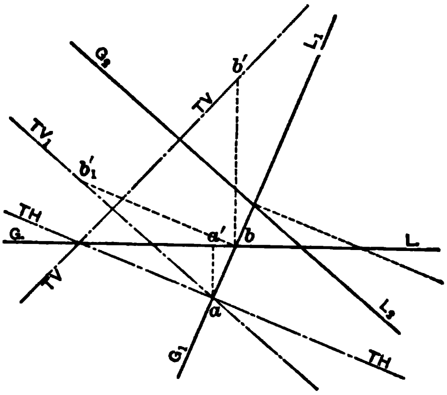

FIGURE 24.

Consider another case of change of coordinate planes, with respect to any oblique plane. Let T, Fig. 24, be the given plane and let it be required to take a new V_1 plane perpendicular to it and a new H_1 plane parallel to it. The G_1 L_1 of V_1 is perpendicular to TH, for if a plane is perpendicular to V its H trace will be perpendicular to the G. L. By preceding paragraph we find TV_1. The new H_1 plane, to be parallel to T must have its $G_2.L_2.$ parallel to the V_1 trace of the plane or TV_1. The plane H_1 cuts a line from H which is perpendicular to V_1 but this perpendicular by construction will be parallel to TH, meeting it at infinity. Hence since TV_1 is parallel to G_2 L_2, TH_1 will also be parallel to and at an infinite distance from $G_2.L_2$.

51. To draw a line parallel to a given plane through a given point.

Analysis:—*From Theorem V, a line is parallel to a plane when it is parallel to a line lying in the plane. There can be an infinite number of lines through a point parallel to a plane, each will be parallel to a line in the plane, hence draw any line lying in the given plane and through the point draw the projections of a line parallel to it.*

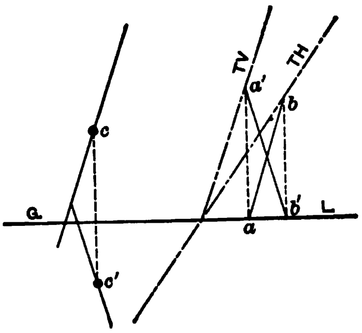

FIGURE 25.

Construction:—Let T, Figure 25, be an oblique plane and C the given point. Draw any line AB in T (whose traces will lie in the traces of the plane). Through *c* draw a line parallel to *ab* and through *c'* a line parallel to *a'b'*. The line through C is parallel to the plane T.

Since an infinite number of lines can be drawn through C and parallel to T, they will constitute a plane, so if two lines are drawn through C parallel respectively, to two lines lying in T, they will lie in a plane parallel to T through the given point. But such a plane, by theorem VI, would also have its traces parallel to those of T, hence it is not necessary to draw but one line through C to locate it.

52. To pass a plane through one line and parallel to another line.

Analysis:—*By Theorem V, a line is parallel to a plane when it is parallel to a line lying in the plane. Hence draw through a point in the first line, a line parallel to the second. The plane of the two intersecting lines will be parallel to the second line.*

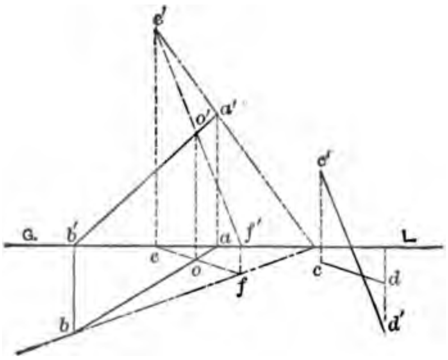

FIGURE 26.

Construction:—In Figure 26, let AB be a line through which to pass a plane parallel to the line CD. Through any point of AB as O draw an auxiliary line EF parallel to CD. The plane of AB and EF is the plane required.

If the two given lines are parallel to each other, i.e., their projections respectively parallel then *any* plane through the one will be parallel to the other.

If either one of the two given lines is parallel to the G. L., the required plane will be parallel to the G.L. and its traces will be parallel to the G.L.

If either one or both of the lines have their projections perpendicular to the G.L., then the traces are obtained by use of an end plane.

There can be but one plane containing a given line and parallel to another line unless the two lines are parallel to each other.

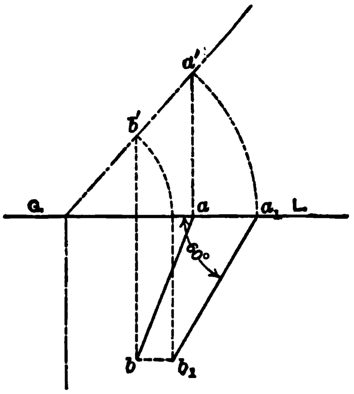

FIGURE 27.

53. Within any given plane perpendicular to a coordinate plane, to draw a line making a given angle with that coordinate plane.

Analysis:—*By Theorem II, one projection of the line will coincide with the trace of the plane, that projection on the coordinate plane to which the given plane is perpendicular; hence we can assume that projection at will. Revolve the assumed projection together with the trace of the given plane into the corresponding coordinate plane about the corresponding trace of the plane as an axis. The angle can here be constructed its true value. Revolve the line back again to its original position, and both projections of the line and the angle will be shown.*

Construction:—Let Fig. 27, show a plane perpendicular to V. Required a line AB lying in it and making an angle of 60° with V. Assume *a'b'* with A a point in V.

Revolve AB about TH as an axis until A fall into H. The angle at a_1 with the the G.L. as one side will be the true angle and b_1 can be obtained in the manner shown. Revolve the line back again into its original position. a_1 moves along the G.L. as in the figure to a at the intersection with a perpendicular from a' to the G.L. and b_1 moves parallel to the G.L. to the position b on a perpendicular to the G.L. through b', whence ab is the H projection of the line.

If the plane T has but one trace, which would of course be parallel to the G.L. then one projection of the required angle would coincide with the given trace of the plane and the other projection of the angle would at once show its true value.

If the given plane is perpendicular to both coordinate planes the angle is obtained by revolving the given plane into V about its V trace. The projection of the angle on the end plane would sbow its true value.

54. Through a given point to pass a plane parallel to two given lines.

Analysis:—*A plane is parallel to a line when it contains a line parallel to the given line. Therefore through the given point draw lines parallel respectively to the given lines, the plane of these lines is the required plane.*

Construction: Let O Fig. 28 be the given point, AB and CD respectively, the given lines. Through O draw two lines, parallel respectively to AB and CD. The V traces of these lines are respectively A_1 and D_1 and the H traces, B_1 and C_1, A_1 and D_1 therefore, are points in the V trace of the plane of the two lines and B_1 and C_1 are points in the H trace, which constitute the plane required.

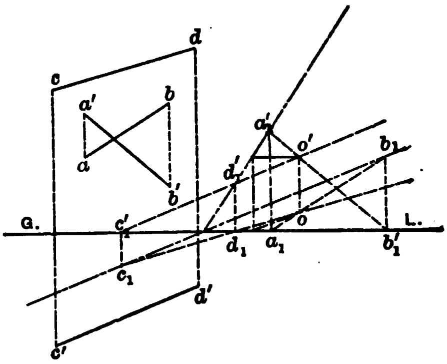

FIGURE 28.

If the given point is in the G. L. the traces of the required plane intersect each other in this point and hence after drawing the two auxiliary lines through the point, draw a third auxiliary line lying in the plane of the two, i. e., intersecting them; the traces of this line will lie on the traces of the required plane thus giving, together with the given point, two points in each trace of the plane.

If it is found that any third auxiliary line just mentioned also has both of its traces in the G. L. then the traces of the required plane are in the G. L.

If both given lines are parallel to the G. L. the required plane will be parallel to the G. L. It is necessary either to use a third auxiliary line as in the case just preceding or an end plane whereon the traces of the auxiliary lines will establish the trace of the required plane with the end plane and from this its traces with V and H may be obtained.

55. To find the line of intersection of two planes;

Analysis:—*Since the required line is a line of each plane its traces will lie in both planes respectively, i.e. its*

V trace will be at the point of intersection of the V traces of the given planes and its H trace at the point of intersection of their H traces. Hence the line connecting these points will be the line required.

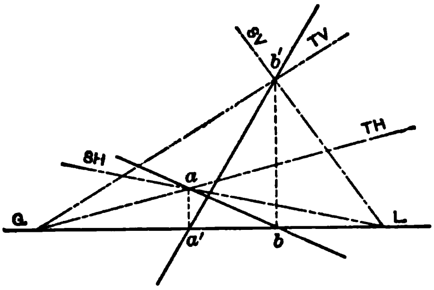

FIGURE 29.

Construction:—Let the two planes be given as in Figure 29. The point of intersection of the H traces is A and it is the H trace of the line of intersection. The point of intersection of the V traces is B and it is the V trace of the line of intersection, hence AB is the line required.

If the traces of both planes do not intersect at accessible points upon the drawing, auxiliary construction must be resorted to as follows:

56. To find the line of intersection of two planes when one or both pairs of traces do not intersect at accessible points.

Analysis:—*Any plane parallel to a coordinate plane will cut from each plane a line parallel to that coordinate plane. These lines, lying in each plane must intersect upon the line of intersection of the two given planes. Hence, pass one or more auxiliary planes parallel to H or V and find two pairs of lines cut from each plane. The* *points of intersection of these lines will determine the required line.*

Construction:—Let the planes T and S be given as in

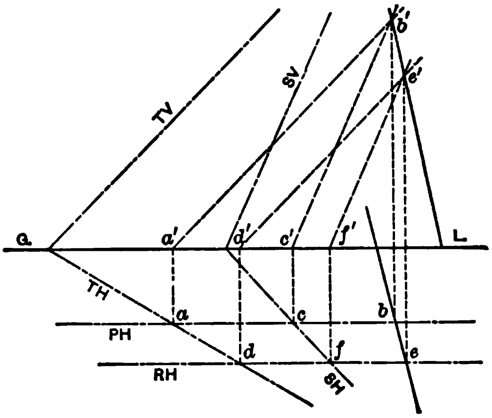

FIGURE 30.

Figure 30, neither pair of traces intersecting at accessible points. Pass any two auxiliary planes P and R parallel to V. They will cut lines AB and CB respectively from the planes T and S, intersecting each other in the point B and lines DE and FE respectively from the planes T and S intersecting each other in the point E. The line BE is the required line of intersection of the two planes.

If only one pair of traces intersect at an inaccessible point, the following convenient construction may be employed:

Analysis:—*Parallel planes intersect a third plane in parallel lines, hence draw an auxiliary plane parallel to one of the given planes and it will intersect the other given plane in a line which is parallel to the required line.*

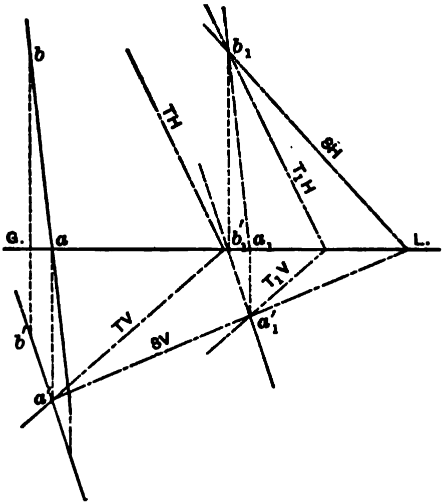

FIGURE 31.

Construction:—Let the planes S and T be given as in Figure 31. At any point upon the G.L. draw a plane T_1 parallel to T, its traces will be parallel to T and by Sec. 47, A_1B_1 will be its line of intersection with S. Now *a'* and *a* are the projections of the V trace of the line of intersection of S and T, hence through A draw the projections of a line parallel to A_1B_1 and it will be the required line.

If both of the given planes cut the G.L. at a common point, either of the two preceeding special constructions may be employed, noting that the line of intersection will pierce the G.L. at the same point as the planes.

If both of the given planes have traces parallel to the G.L., their line of intersection will be parallel to the G.L. and the intersection of the planes with an end plane is necessary to determine the distance of this line from the coordinate planes in both projections. Its projection on the end plane will be the point of intersection of the traces of the given planes with the end plane.

When two given planes are parallel, their line of intersection is at infinity and its projections are at infinity in both coordinate planes.

When two planes have one pair of parallel traces, the line of intersection will have its similar projection parallel to the parallel traces, the corresponding projection will be parallel to the G.L.

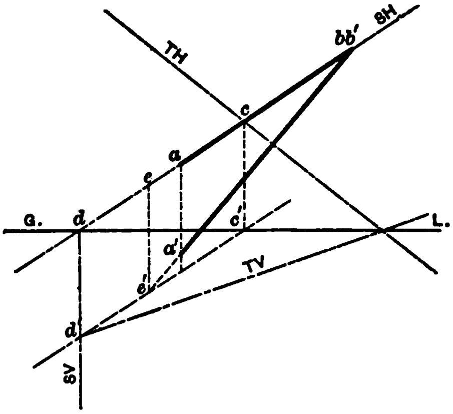

FIGURE 32.

57. To find the trace of a line with any plane.

Analysis:—*The required trace will lie upon the line of intersection with the given plane of any plane passed through the given line, hence pass any auxiliary plane through the line, find its line of intersection with the given plane and where the given line pierces this line of intersection will be the trace required. The most convenient auxiliary plane for this purpose is either the V or the H projecting plane of the line.*

Construction:—Let T in Figure 32 be any plane and AB any line oblique to the plane T, to H and to V. Draw the H projecting plane, S, of the line AB. Find its line of intersection, CD, with the plane T. Note that the H projection of this line will be coincident with the H trace of the plane S, since S is perpendicular to the H plane. Where CD intersects the line AB in E is the trace required.

If the plane is oblique and the line is parallel to the G.L., either projecting plane of the line will have one trace at infinity and the line of intersection of the projecting plane and the given plane will have one projection parallel to the similar trace of the given plane, the corresponding projection will be parallel to the G.L.

If the given plane is an end plane, the point of intersection of the projections of the line with the traces of the plane will be the projections of the point of intersection of the line with the plane.

If the given plane is perpendicular to one coordinate plane, and oblique to the other, while the line is oblique to both, then note that every point in the given plane is projected upon that plane to which it is perpendicular in its trace upon that plane. Hence one projection of the piercing point will be where the one projection of the line cuts the trace of the plane with the coordinate plane to which the latter is perpendicular.

If the given plane is oblique and the given line is perpendicular to the G.L., then the projecting planes of the line coincide and are together an end plane. Find the line of intersection of the given plane with the end plane and the piercing point will be on this line.

58. THEOREM VII.

If a line is perpendicular to a plane the projections of

the line are perpendicular respectively to the traces of the plane.

> **Proof:—The projecting plane of a line, whose one trace is identical with the projection of the line, is perpendicular to the coordinate plane, and is also perpendicular to the given plane since it contains a line perpendiclar to the given plane; hence it is perpendicular to the line of intersection of the two; but this line of intersection is the trace of the given plane with the coordinate plane, hence, either projection of the line will be perpendicular to the similar trace of the plane.**

59. Through a given point to pass a plane perpendicular to a given line.

Analysis:—*By Theorem VII, the traces of the required plane are perpendicular respectively to the projections of the line. Therefore through the given point draw a vertical or a horizontal of the required plane since one projection of which will have the same direction as the similar trace of the required plane. The trace of this line upon the corresponding plane of projection will be one point in the corresponding trace of the required plane. Both traces of the required plane can then be drawn since they must intersect upon the G.L.*

Construction:—Let AB, Figure 33, be a given line and O the given point. Through O draw the line OP as a horizontal of the required plane, i.e., *op* is perpendicular to *ab* and *o'p'* is parallel to the G.L. The V trace, C, of the line OP is a point in the vertical trace of the required plane and the H trace goes through the point in which the

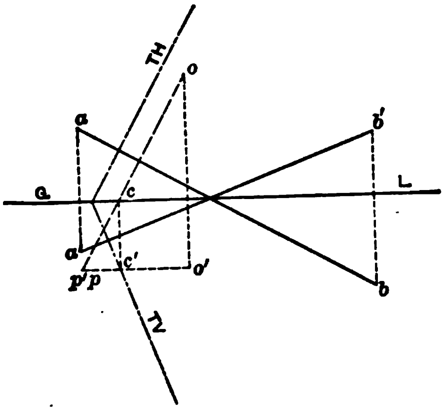

FIGURE 33.

V trace touches the G.L., both traces being perpendicular respectively to the projections of AB.

If the given point is in the G.L., the traces of the required plane will intersect each other in this point.

If the given line is in an end plane and the given point is in the G.L., then the traces of the required plane are in the G.L., and the plane is indeterminate in position. It will be completely determined by ascertaining the piercing point of the given line with it.* This is done by projecting both plane and line upon an end plane since the line is parallel and the plane is perpendicular respectively to the end plane. The point in which the end projection of the line cuts the end trace of the plane is the end projection of the piercing point of the line with the plane.

If the given line lies in an end plane and the point is outside of the line then the required plane is parallel to the G.L., and contains a line through the given point projectively perpendicular to the given line.* Hence to find the traces of the plane, project the line and point upon an end plane. Draw through this projection of the point the end projection of a line perpendicular to the end projection of the given line. The traces of this line lie upon the traces of the required plane.

---

* A plane whose traces are in the G.L. is completely determined by the location of a point in the plane.

60. To find the distance of a point from a plane.

Analysis:—*The distance of a point from a plane is the length of a perpendicular from the point to the plane. Hence draw through the point a line perpendicular to the plane. Find the piercing point of this line with the plane. [by Sec. 57]. Next find the distance between the two points on the line by any of the methods previously explained.*

Construction;—Let T, Figure 34, be a given plane and O a given point. Through O draw OR the indefinite perpendicular to the traces of the plane T. By means of the H projecting plane S of the line OR, and its line of intersection with the plane T, obtain the piercing point P. The length of the line connecting O and P can then be obtained by revolving the line about O as a center until it is parallel to H whence op_1 is the length required.

If the given plane is an end plane, the perpendicular distance required is a line parallel to both H and V, and hence is projected its true length in the line drawn through the projection of the point and perpendicular to the traces of the plane.

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* Projectively perpendicular means that the projections on the plane are perpendicular.

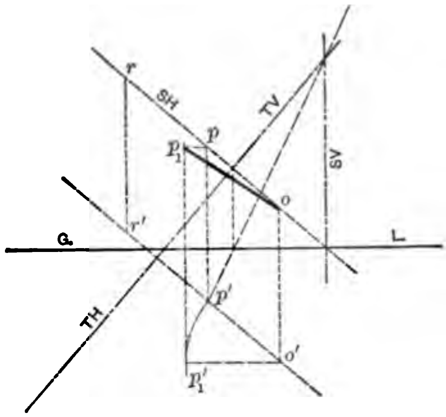

FIGURE 34.

If the given plane is parallel to the G.L., the perpendicular distance required is a line lying in an end plane, hence to find its length project both plane and point upon an end plane whence the distance between the end projection of the point and the end trace of the plane is the required distance.

61. Definitions;—

From geometry we have by definition:—*a line is perpendicular to a plane when it is perpendicular to every line in the plane through its foot;* or, in other words, all of the perpendiculars to a straight line at a given point in it lie in a plane perpendicular to the line; therefore, while a line, which is perpendicular to a plane, has its projections perpendicular respectively to the traces of the plane, two lines which are perpendicular to each other do not necessarily have their projections perpendicular. For by reference to Figure 34, any line in the plane T and passing through the piercing point of the line OR with the plane T would be perpendicular to the line OR. Or again, any line in the plane T would be parallel to some line which could be drawn through the piercing point of the line OR with the plane T and this latter line would be perpendicular to OR because it lies in a plane perpendicular to OR. Much confusion is apt to arise upon these two points.

The angle a line makes with a plane is the angle which it makes with its projection on *that* plane.

The angle between two planes may be defined as the plane angle between two lines lying one in each plane, and perpendicular to the line of intersection of the planes at a common point. These lines are also known as the lines of *greatest declivity* of the one plane with respect to the other.

62. THEOREM VIII.

If two lines are perpendicular to each other in space and one of them is parallel to a coordinate plane of projection their projections on that plane are perpendicular to each other.

Proof:—For since one of the lines is parallel to a coordinate plane, the projecting plane of any other line perpendicular to it will have its trace with the coordinate plane perpendicular to the projection of the first line, but this trace is the projection upon the coordinate plane of the second line, hence the projections of the lines upon the one coordinate plane are perpendicular to each other.

63. To find the angle between two lines which intersect.

Analysis:—*The value of the angle appears when the plane of the intersecting lines is either parallel to or is coincident with a coordinate plane; hence if the lines are oblique to both coordinate planes, revolve the vertex of their angle about a trace of the plane of the lines until it lies in the coordinate plane of that trace. The traces of the given lines are fixed points in the revolution, hence are points in the sides of the revolved position of the angle, whence the true angle can be obtained.*

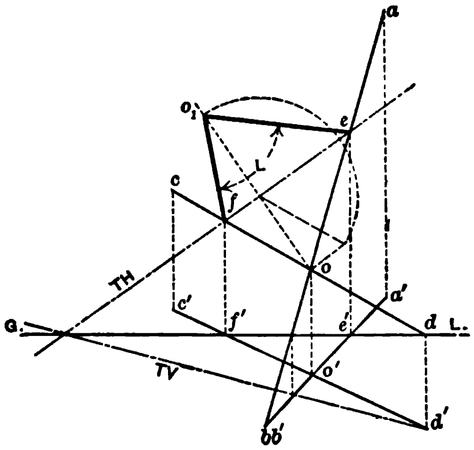

FIGURE 35.

Construction:—Let AB and CD, Figure 35, be two lines intersecting in the point O. Find the plane T of the two lines. E and F are the respective H traces of the lines lying on the H trace of their plane. Revolve the vertex O about the H trace of this plane into H. The H traces of the lines are fixed points in the revolution since they lie on the axis. o_1 is the revolved position of the vertex and eo_1f or its supplement will be the required angle.

If one of the lines is parallel to a coordinate plane, the most convenient construction is to revolve the other line about it until it is parallel to the same plane, whence the angle appears its true value as projected on that plane.

64. To draw the projections of any desired division of an angle between two lines.

Analysis:—*Find the true value of the angle between the two given lines by Section 63. Then draw the revolved position of any radial line dividing the angle into desired proportional parts. The intersection of this line with the axis of revolution or trace of the plane of the lines is a fixed point. Connect this with the projection of the vertex of the angle whence the angle is divided in projection.*

65. Through a point in a given oblique plane to draw two lines making a given angle with each other.

Analysis:—*Revolve the given point as the vertex of the angle about either trace of the plane until it lies in the coordinate plane of that trace. Draw two lines through this revolved position of the vertex, to intersect the trace of the plane in two fixed points in the sides of the projection of the required angle. Connecting these points with the projection of the vertex gives one projection of the angle.*

66. To project a line on any oblique plane.

Analysis:—*The projection of a line on a plane is the locus of the projections of all points of the line upon that plane. Hence take any two points of the line and draw perpendiculars to the plane (Theorem VII) and find their piercing points (by Section 57). The line joining these piercing points will be the projection desired.*

If the given line pierces the given plane within the limits of the drawing it is only necessary to project one other point of the line on the plane since the piercing point is one of the desired points in the projection.

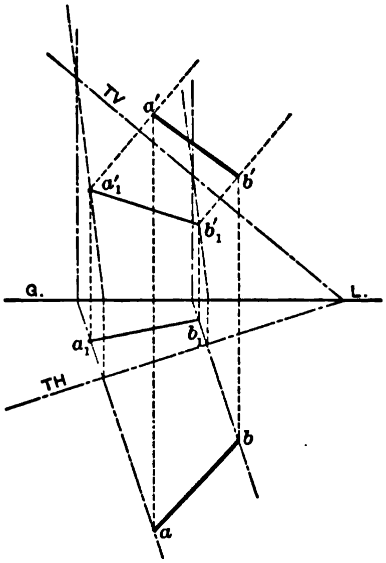

FIGURE 36.

Construction:—Let AB, Figure 36, be the line and T the plane. From A and B drop perpendiculars AA_1 and BB_1, respectively, to the plane T. Find the piercing points of these perpendiculars, by using their H projecting plane in each case. A_1 and B_1 will be the respective piercing points whence $a'_1 b'_1$ is the vertical projection, and $a_1\ b_1$ the horizontal projection, of the given line on T.

If the given plane is perpendicular to either coordinate plane, then the trace on that plane to which it is perpendicular will contain at once the piercing points of the perpendiculars from the points in the given line to the plane.

If the given plane is parallel to the G.L., it is necessary to use an end plane, for the projecting perpendiculars of the line lie in planes perpendicular to the G.L. The intersections of perpendiculars from the points on the end projection of the line to the end trace of the given plane, are the projections of the respective piercing points with the plane.

If the given plane has its traces in the G.L., an end plane is also necessary and for similar reasons.

67. To find the angle a line makes with a given plane.

Analysis:—*Since by definition the desired angle is that between the line and its projection on the plane, project the given line upon the given plane, then find the value of the angle by revolving the line and its projection about either trace of their plane with the coordinate plane into the latter, whence the true value appears.*

Construction:—Let the line AB and the plane T be given as in Figure 37. Find the projection A_1B_1 of AB on the plane T by Sec. 66. Find the V trace of the plane of the lines AB and A_1B_1. The V trace of AB is C and that of A_1B_1 is D. Revolve A and A_1 about the V trace of the plane of AB and A_1B_1 into V. A falls at e' and A_1 falls at e'_1. Hence the angle required is that between $e'c'$ and e'_1d'.

If the given plane is parallel to the G.L. or has its traces in the G.L., it is necessary to use an end plane to get the projection of the line upon the given plane.

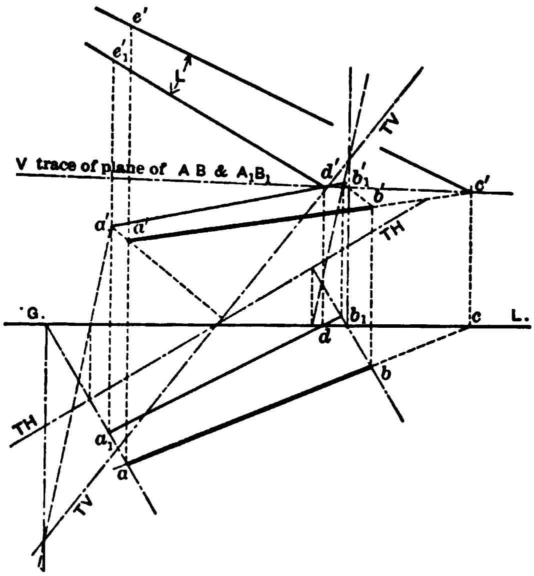

FIGURE 37.

68. To find the angle between two planes.

Method No. 1. Analysis:—*By definition, if a plane is passed perpendicular to their line of intersection, it will cut a line from each which is a measure of the angle required. Therefore assume a point upon the line of intersection and find the traces of a plane perpendicular to the latter and containing the point. The points in which either trace of this auxiliary plane cuts the corresponding traces of the given planes will be points in the sides of the required angle. The assumed point on the line of intersection of the planes is the vertex of the angle. By revolution into a coordinate plane, the true value can be obtained.*

Construction:—Let two planes T and S be given as in Figure 38. Assume the point O upon their line of inter-

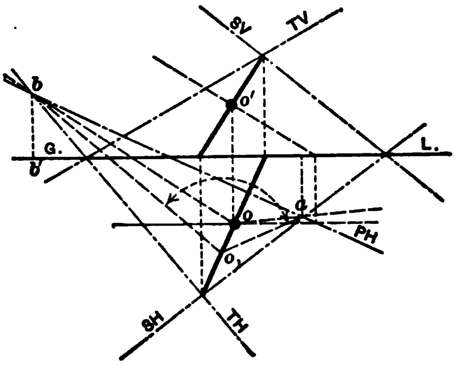

FIGURE 88.

section through which to pass a perpendicular plane. By use of a vertical of the plane its H trace, PH is found, and from the intersections of PH with the H traces of S and T, the points A and B establish the sides of the angle AOB. The vertex O revolved into H about PH falls at o_1 whence ao_1b is the true value of the angle required.

To assume the point upon the line of intersection through which to pass a perpendicular plane does not always bring a trace of the plane upon a convenient part of the drawing to construct the angle, hence while this method is simple to understand it is not in general the best.

Method No. 2. Analysis:—*An auxiliary plane that is perpendicular to the line of intersection of the two given planes and cuts from each plane a line, cuts also a line from the projecting plane of the line of intersection which in turn pierces the coordinate plane in the trace of the auxiliary plane with the latter. Hence a trace of the auxiliary plane may be assumed and the piercing point with it found of the line of intersection of the two given planes. The* *remainder of the solution is the same as for Method No. 1.*

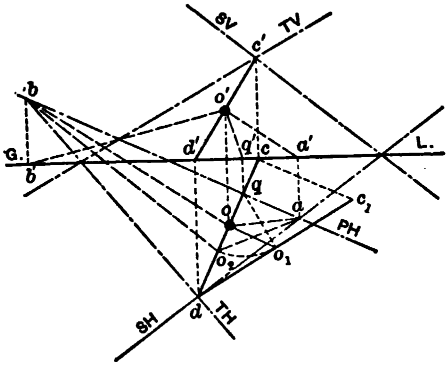

FIGURE 39.

Construction:—Let the conditions be given again as they were in Figure 38, see Figure 39. Assume the H trace of an auxiliary plane P perpendicular to the line of intersection of the planes.* The point *q* in which this trace cuts the H projection of the line of intersection is the H projection of the H trace of the line OQ cut from the projecting plane of the line of intersection by the auxiliary plane. Revolve the line of intersection CD about its H projection into H and at *q* draw a perpendicular to it qo_1. O is the piercing point of the line CD with the auxiliary plane P and *o* is its H projection. Whence *aob* is the projection of the angle. By revolving the vertex O into H at o_2, it is found that ao_2b is the *true value of the angle.*

If both planes are perpendicular to a coordinate plane, the angle between their traces on that plane is the angle between the planes.

If one of the planes has its traces in the G.L., or if both planes have their traces in or parallel to the G.L., then it is necessary to find their intersection with an end plane. In the first case the intersection of the traces of both planes on the end plane is a point in the line of intersection of the two planes. In the latter two cases the angle between the traces on the end plane is the true angle between the planes.

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*Its corresponding trace is not needed in the construction.

If the traces of both planes intersect each other on the G.L., it is necessary to either use auxiliary planes perpendicular to a coordinate plane or to pass an auxiliary plane parallel to one of the given planes in order to get the projections of the line of intersection of the planes. In the latter case the projections first obtained are those of a line parallel to the line of intersection.

If the angle between any oblique plane and a coordinate plane is desired. the auxiliary plane will be perpendicular to the coordinate plane and hence cut from the oblique plane a line of greatest declivity with respect to the coordinate plane and from the latter a line of greatest declivity with respect to the oblique plane, the angle between which is the desired angle.

69. Given a line in a plane, to draw another plane intersecting the first in the given line and making a given angle with the given plane.

Analysis:—*The given line will be the line of intersection of the two planes, hence to establish the plane angle which is given, through any point of the given line pass a plane perpendicular to it and note its intersection with the given plane. Revolve the line of intersection about a trace of the auxiliary perpendicular plane into a coordinate plane and construct the angle required. Revolve it back again and* *the second side of the angle as derived and the given line determine the plane desired.*

70. To find the angle between two planes by the method of change of coordinate planes.

Analysis:—*If one coordinate plane is taken parallel to the line of intersection of the given planes and the corresponding one perpendicular to it, this line will be projected upon the latter coordinate plane as a point and the angle between the planes will be projected upon the same plane, its true value, being the angle between the traces.*

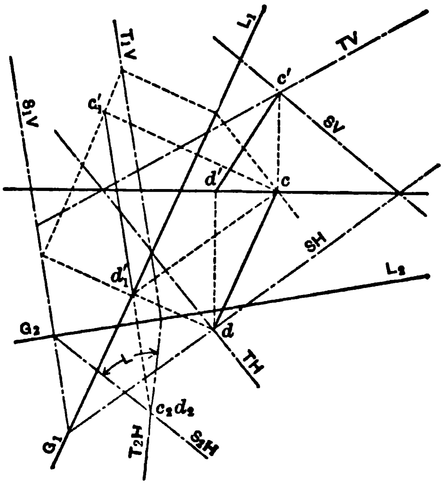

FIGURE 40.

Construction:—Let the planes T and S be given as in Figure 40. Take a $G_1.L_1$. parallel to the H projection of the line CD, which is the line of intersection of the planes, giving $c'_1d'_1$ as the V projection of the line on the V_1 plane parallel to it; on this plane also find the V traces of T and S by using C as a point in each plane and drawing horizontals of the planes respectively. T_1V and S_1V are the traces so found. Next take a $G_2.L_2$. perpendicular to the V_1 projection of CD getting the new H_1 projection c_2d_2 as the projection of the line on this plane. This is also a point in the new H traces of the planes T and S. Hence the angle between T_2H and S_2H is the true angle between the planes.

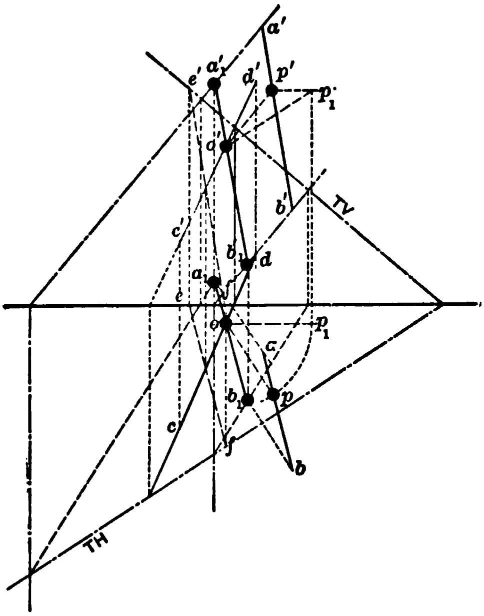

FIGURE 41

71. To find the common perpendicular of two non-intersecting lines.

Analysis:—*If an auxiliary plane is passed through one of the lines and parallel to the other the common perpendicular will be projected on this plane as a point, i.e., the point of intersection of the one line and the projection of the* *other. Hence at this point erect a perpendicular to the auxiliary plane to meet the other line. It will be the line desired.*

Construction: Let the lines AB and CD be given as shown in Fig. 41. Pass a plane through the line CD and parallel to AB, (by Sec. 52) using an auxiliary line EF parallel to AB and intersecting CD. TH and TV are its traces. Project AB on the plane T by using perpendiculars from A and B to the plane T (by Sec. 66) their piercing points are A_1 and B_1 respectively. A_1 B_1 is the projection of AB on the plane T. At O, the point of intersection of the projections, draw the projections of a perpendicular to the plane T. It intersects the line AB in the point P. OP is the required perpendicular $o'p_1'$, is true length of this perpendicular.

Method No. 2, Analysis:—*By change of coordinate planes, project the lines upon a new V_1 plane which is parallel to one of them and upon a new H_1 plane perpendicular to the same line. The required distance is the perpendicular distance between the two lines as projected on H_1, one of them being projected as a point.*

72. Given one trace of a plane and the angle the plane makes with that coordinate plane, to find the other trace.

Analysis:—*If the trace of an auxiliary plane is drawn which cuts out the lines of greatest declivity of the coordinate plane and the given plane with respect to each other, its trace will be perpendicular to the given trace and the projections of the lines cut out of each plane will coincide with it. Hence assume a line of greatest declivity of the given plane to be revolved about its projection on the coordi-*

nate plane, until it lies in the latter. It will then make the angle with its projection that the required plane is to make with the coordinate plane. Construct this angle and find the revolved position of the piercing point with the other coordinate plane, of the line of greatest declivity. The true projected position of this piercing point is a point in the desired trace.

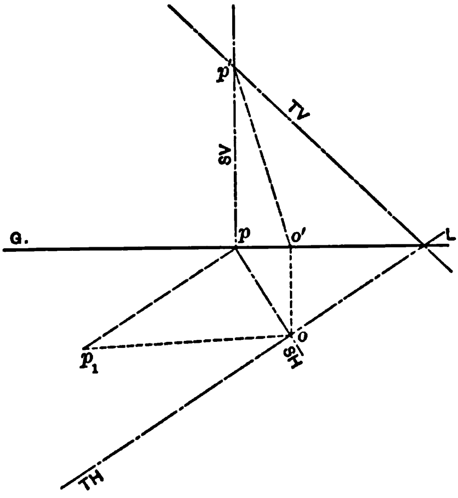

FIGURE 42.

Construction:—Let the trace TH of a plane be given as in Fig. 42 and let its required angle with H be 60°, SH is the trace of any auxiliary plane perpendicular to both T and H. The portion of it, *op*, coincides with the H projection of the line cut from the plane T and also that of the line cut from H. Assuming the former to be revolved into H, it falls at op_1, making an angle of 60° with *op*. The point p_1, being on a perpendicular from *op* to the line op_1, is the revolved position of the piercing point of the line of greatest declivity of the plane T with the V plane, hence the projection of the piercing point is a distance above the G.L. on a perpendicular at p that the point p_1 is from p. TV is the required trace of the plane.

73. To find the distance from a point to a line.

Method No. 1, Analysis:—*If a plane is passed through the point perpendicular to the line, the distance from the given point to the piercing point of the line, with this plane, will be the required distance.*

Method No. 2, Analysis:—*Pass a plane through the point and the line, revolve both about either trace of their plane until they lie in the coordinate plane; the perpendicular distance between point and line will be projected its true length.*

74. Given one trace of a plane and the angle it makes with the corresponding plane of projection, to find the corresponding trace.

Analysis:—*The plane will be tangent to a right circular cone whose apex is on the given trace with its base in the corresponding coordinate plane and axis perpendicular to the latter; the elements of this cone will make the same angle with the corresponding coordinate plane that the required plane makes. Hence assume any point upon the given trace and draw the projection of the cone upon that coordinate plane;-it will be an isosceles triangle with base in the G.L., and base angles equal to the given angle. Draw the corresponding projection of the cone which will be a circle. The trace of the required plane upon this corresponding plane will be tangent to this circle.*

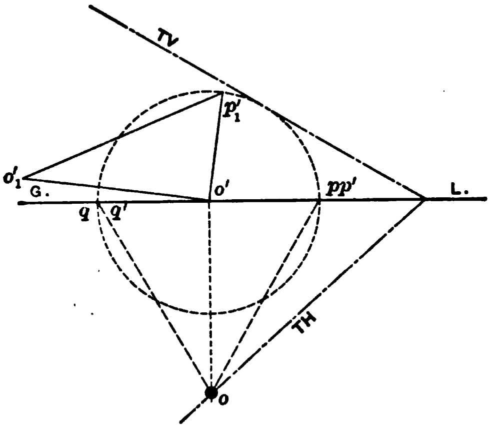

FIGURE 43.

Construction:—Let TH be given as in Figure 43 and let the required angle with V be 60°. Assume any point on it as O. Draw OP and OQ the contour elements of the projection of the cone on H whose *base* angles are 60°. qp is the H projection of the base in V. Draw a circle with o' as a center and $o'p'$ as a radius. Tangent to this circle draw TV, the required trace.

Note:—The problem could be solved by considering only the triangle formed by a plane passed perpendicular to V and the plane T. $o'o'_1$ is the revolved position of the line cut from H and could be drawn from o' in any direction. $o'p_1'$ is also the revolved position of the line cut from V and will be perpendicular to $o'o'_1$. Lastly, the line $o'_1p'_1$ is the revolved position of the line cut from the plane T and can be drawn to make the required angle with $o'p'_1$. The required V trace, then, is tangent to a circle of radius $o'p_1'$ The true revolved position of the triangle is when op'_1 is perpendicular to TV.

75. Given the angles a plane makes with both coordinate planes to locate its traces.

Analysis:—*The required plane can be considered as tangent to a sphere of any radius whose center is in the G.L. If planes are assumed which are perpendicular to the given plane and to the coordinate planes respectively, thus cutting out lines from each which will be measures of the angles the required plane makes with the respective coordinate planes, and if further these planes are passed through the center of the sphere they will cut lines respectively from the required plane which are tangent to the sphere. Therefore the triangles formed by the assumed planes cutting the required plane and both coordinate planes in each case can be assumed separately—dealt with as the one triangle was dealt with in Section 72. Observing the fact that the revolved position of the lines cut from the required plane in each case will be tangent to a circle which is the projection of the sphere and have each one vertex at the center of that sphere.*

Construction.—Let the required angles of a plane with V and H be α and β respectively. See Fig. 44. Draw the projections of a sphere with any radius and center at any point O. Next draw an assumed revolved position of the lines which would be cut from the V plane, the required plane and the H plane, by a plane assumed to be perpendicular to the first two. $o'a'b_1$ is the triangle made by these lines. oa' is the line cut from V, $a'b_1$ the line cut from the required plane and $o'b_1$ the line cut from the H plane. $b_1a'o'$ is made equal to α and $a'b_1$ is drawn tangent to the sphere. The V trace of the plane is tangent to a circle with radius $o'a'$ and the H trace goes through a point b on

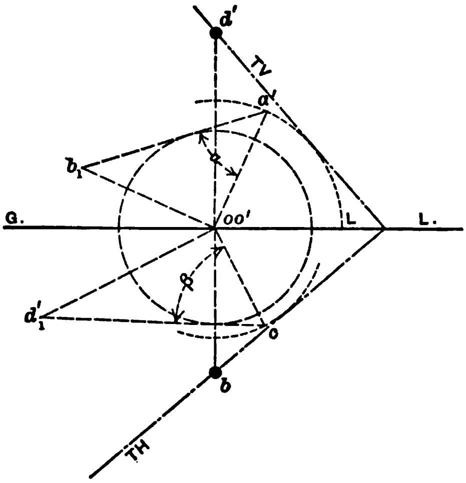

FIGURE 44.

a perpendicular to the G.L. at O, and distant from O equal to ob_1. Likewise, and constructed similarly, ocd'_1 is the triangle formed by lines cut from the H plane, the required plane and the V plane by a plane perpendicular to the first two, with the angle β as shown. Oc is the line cut from H, cd'_1, the line cut from the required plane and Od'_1, the line cut from V. The H trace of the required plane is tangent to a circle of radius oc and the V trace goes through the point d', a distance od'_1 above H.

76. Given the angle a line makes with both coordinate planes to draw its projections.

Note:—In this problem the analysis for the general solution is difficult to follow hence a special solution is added after the general form.

General Analysis:—*Consider the line as limited by its* *traces. Assume either projecting plane of the line to be revolved into either coordinate plane about its intersection with that coordinate plane. Revolve the other projecting plane of the line bodily with it into the same coordinate plane and about the line of intersection of the two projecting planes, which is the line itself. The revolved position of the line is the hypotenuse of two right angled triangles, an angle of each of which being oppositely directed, is the respective angle the line makes with the coordinate planes.*

Specific Analysis:—*Assume a line limited by its V and H traces. Revolve the H projecting plane, bounded by the line, its H projection and the V trace of the projecting plane, into V; next revolve the V projecting plane of the line bounded by the line, its V projection and the H trace of the projecting plane, into V about the revolved position of the line when it lies in V. In the first triangle, the angle between the G.L. and the revolved position of the line is the angle with H. Opposite to that, with the revolved position of the line as a side, is the angle the line makes with V, from which, by completing the triangle, the distance of its H trace from the G.L. can be found. Next revolve back into proper projective position the first triangle, to do which the H trace must be located at the distance from the G.L. which it is found to be from the revolved position of the second triangle.*

Construction: Assume $oa'b_1$, Fig. 45, to be the revolved position of the H projecting plane of a line of which $a'b_1$ is the line itself. On the latter as an axis, assume the V projecting plane to be revolved into V. The perpendicular distance of c'_1 from $a'b_1$ is the revolved position of the distance of the H trace from the V plane, i. e.

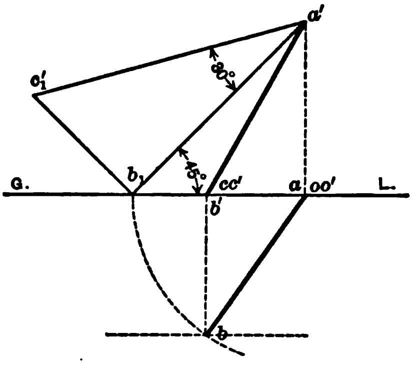

FIGURE 45.

from the G. L. Next, lay off the distance $c'_1\ b_1$ on H from the G. L. and draw a horizontal line. Where an arc of a circle, with radius ab_1 and center at a cuts this horizontal in b, is the H projection of the H trace of the line and ab is the H projection of the required line. The V projection is $b'a'$.

77. To draw a regular pyramid of given base and altitude with its base in a given oblique plane.

Analysis:—*(1.) Assume the base as revolved about either trace of the given plane until it lies in the coordinate plane of that trace, where it can be drawn its true shape. Revolve it back again into the given plane by any convenient method. Next, to find the projections of the apex of the pyramid, draw a perpendicular to the plane at the center of the base, and by any convenient method, get the projected distance laid off on this perpendicular, i.e., the projection of the altitude of the pyramid. The apex connected with the corners of the base completes the pyramid.*

Analysis:—*(2.) Assume the pyramid to be projected* *upon a plane perpendicular to that of the base, i.e., parallel to the axis. Assume this projection as revolved about the trace of its plane with the coordinate plane until it lies in the latter. Here the axis will appear its true length, since it lies in the coordinate plane the base will be projected as a straight line. By revolving, next, the base about its intersection with the coordinate plane into the latter, its true shape will appear and the relative position of its vertices be ascertained. By counter revolution the projection of the pyramid may be constructed.*

Construction:—*According to Analysis 2.* Let a hexagonal pyramid with diameter of base of 2 inches and altitude 2¾ inches, have its base in any oblique plane T, see Figure 46, and an edge of the base lying in H. Assume the pyramid to be projected on any plane S perpendicular to H and to T. The line ab_1, is the revolved projection of the base on S, being also the revolved position of the line cut from T by S and p_1 is the apex also revolved into H. Next assume the base of the pyramid to be revolved about ab_1, as a diameter, giving 1_1, 2_1, 3_1, etc., as the vertices of the base 1_{11}, 2_{11}, 3_{11}, etc., are the true projected positions of the vertices on the plane S when the latter is revolved into H. The horizontal distances of the vertices from the diameter are constant, hence the edge of base in H can be assumed anywhere and by counter revolution the vertices 1 , 2 , 3, etc., found. The vertical projection of the pyramid is obtained by noting that the distances of points above the G.L. are respectively equal to the distances of the vertices 1_1, 2_1, 3_1, etc., above SH.

It is to be noted that the method just described is

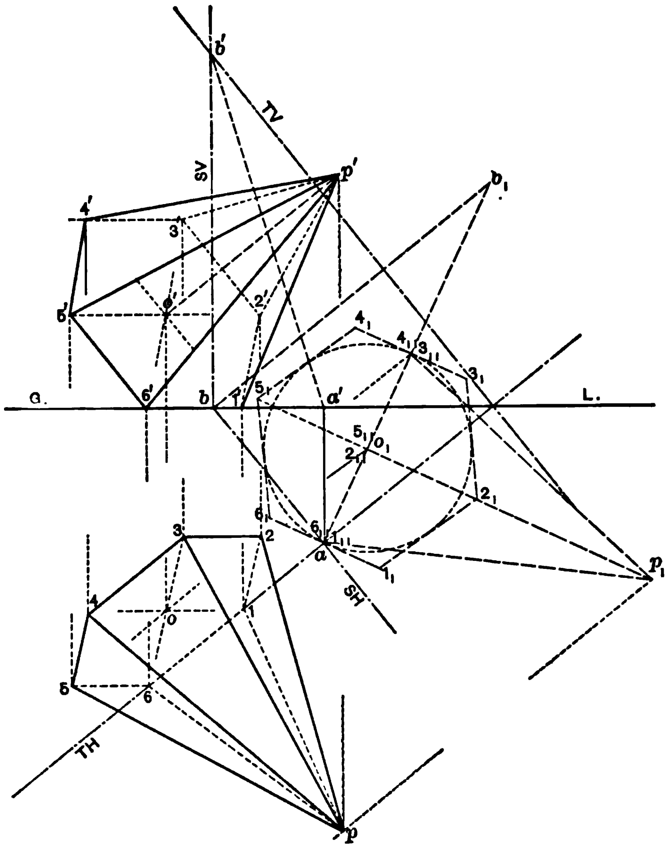

FIGURE 46.

really that of a change of coordinate planes where SH is the G.L. of a new V_1 plane parallel to the axis of the pyramid.

78. To draw a circle through any three given points.

Analysis:—*The three points are the extremities of chords of the circle passing through them. Hence find the plane of* *the three points; revolve them about either trace of their plane into the coordinate plane and draw the circle passing through them. Revolve the circle into its true projective position by any convenient method.*

Note:—A convenient method is to use parallel chords perpendicular to the trace of the plane about which the three points are revolved.

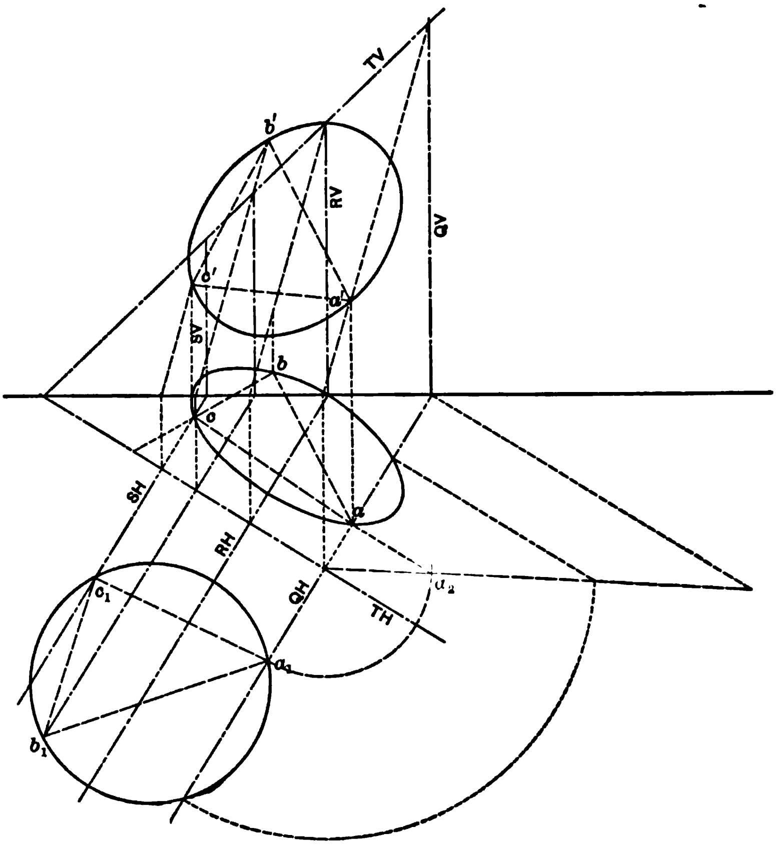

FIGURE 47.

Construction:—Let the three points A, B and C be given as in Figure 47. The traces of their plane are TH and TV respectively. Revolve the points about the H trace of their plane into H to fall at a_1, b_1 and c_1, respectively, whereupon the circle passing through them may be constructed.

To revolve the circle back into its true projective position, the following method may be employed. Pass a series of auxiliary planes Q, R, S, etc., perpendicular to the H plane and the plane T to cut the circle lying in the latter plane, each in two points. The lines of intersection of these several planes with T, will pass through these points, respectively; further, the points revolved into H about TH, will lie on a perpendicular to TH as shown. By counter revolution these may be located on the respective lines of intersection of the auxiliary planes with T.

79. Through a given point, to draw a line making a given angle with a given oblique plane.

Analysis:—*The locus of the lines which can be drawn from the given point to the given plane, making a certain angle with it, form the elements of a right circular cone whose base is in the plane and whose apex is the point. Therefore draw the cone with elements at the required angle with the base by a similar method to that for the hexagonal pyramid in Section 77. Choosing any one element, revolve it back into true projective position. The construction is left to the student.*

80. Through a given line, to pass a plane perpendicular to a given plane.

Analysis:—*The required plane must contain a perpen-*

dicular to the given plane, hence, through any point of the given line, draw a perpendicular to the given plane. The plane of the two lines is the plane required. The construction is left to the student.

81. Locii.

When a point moves so as to satisfy a given condition or conditions, the path it traces is called its locus under these conditions.

As an example, *the parabola* is the locus of a point moving in a plane according to the conditions of the equation $y^2=4px$. This may be defined, in graphical terms, as the locus of a point which moves in a plane so that its distance from a fixed line is equal to its distance from a fixed point. The line is known as the directrix, and the point as the focus of the parabola. The ratio of the two distances is, in mathematics, called the eccentricity (e); for the parabola, it is equal to unity. It has different values for the different conics.

The ellipse is the locus of a point, which moves in a plane, so that the ratio of its distance from a fixed line to its distance from a fixed point is greater than unity, in other words satisfies the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

The hyperbola is the locus of a point which moves in a plane, so that the ratio of its distance from a fixed line to its distance from a fixed point is less than unity, in other words satisfies the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

The circle, defined in the same terms, is the locus of a point, which moves in a plane, so that the ratio of its distance from a fixed line to its distance from a fixed point is equal to infinity. In more generally understood language, it is the locus of a point, which moves in a plane, so as to be equally distant from a fixed point. Graphical locii are a legitimate part of descriptive geometry.

82. To draw the locus of points equally distant from two given planes.

Analysis:—*The locus of points equally distant from two straight lines in a plane is the bisector of the angle between the two lines, hence the locus of points equally distant from two planes, each containing one of the lines, is the plane bisector of the dihedral angle between the two planes. Therefore find the angle between the two planes, bisect it and pass a plane through the bisector and the line of intersection of the planes. This plane will be the required locus.*

83. To draw the locus of points in space equi-distant from three given points.

Analysis: - *The locus of points in space, equally distant from two points, is a plane perpendicular to and bisecting the straight line connecting the two points. Hence the locus of points, equally distant from three points, will be the common line of intersection of three such planes. Therefore find the plane of the three points and by revolving them about either trace of their plane find the perpendicular bisectors of the lines connecting the points; these will meet in a point which is the center of the circle passing through the three points. Find the true projective position of this point in the plane. Through it draw a line perpendicular to the plane of the points which will be the locus required.*

REVIEW QUESTIONS

If two lines are parallel in space, are their projections parallel ?

If two lines are perpendicular to each other in space, are their projections perpendicular ?

If two planes are parallel to each other in space, are their respective traces parallel ?

If two planes are perpendicular in space, are their respective traces perpendicular to each other ?

If a line is parallel to a plane, are its projections parallel respectively to the traces of the plane ?

If a line is perpendicular to a plane, are its projections, respectively perpendicular to the traces of the plane ?

What is the locus of points equi-distant from a given straight line ?

What is the locus of lines which make a constant angle with a straight line at a common point in it ?

What is the locus of points in space equi-distant from a fixed point ?

What is the locus of points in space equi-distant from two intersecting lines ?

CHAPTER IV.

GENERATION AND CLASSIFICATION OF LINES AND SURFACES.

84. A line is mathematically the path of a moving point under fixed conditions, i.e., the locus of the point under those conditions; it may also be an element of a surface which latter also has a mathematical interpretation as the path traced by a moving line under certain conditions.

A line may also be defined, according to various properties of the line, one of which, for example, is the intersection of two surfaces or solids.

A straight line, or right line, is the path of a point which moves in a fixed direction, or again, the intersection of two planes.

Any two consecutive positions of a moving point, being infinitely close, may be regarded as the extremities of an infinitesimal element of the path traced, or an elementary line of it, and in going from the one position to the next, no matter what the ultimate direction of the path, the elementary path traced is straight.

All lines may be grouped into three classes according to the law governing the motion of the generating point. (1.) *Right lines.* (2.) *Single curved lines.* (3.) *Double curved lines.*

If a point moves in a fixed direction, the path generated is a straight line.

If a point moves in a constantly changing direction the path is a curve; whether it be a curve of *single curvature* or a curve of *double curvature* depends upon the following conditions:

If any four consecutive points, that is, points in the curve infinitely near to each other, or any three consecutive elements, that is, right lines connecting the consecutive points as defined, lie in the same plane, then all the points of the path lie in the same plane and the curve is a curve of single curvature, i.e., a plane curve.

If no four consecutive points nor any three consecutive elements, passing through the four points, lie in the same plane then the curve is a curve of double curvature, i.e., it is not a plane curve.

A curve of any kind may be further considered as generated by the motion of a line intersecting itself in successive positions in which case the curve is known as the envelope of the successive positions of the line.

For example, if two intersecting lines as in Figure 48, are divided into a number of equal parts and the division on one farthest from the vertex is joined to the first division from the vertex on the other and so on until all the divisions are connected by lines, it will be seen that these lines are tangent to a curve. This curve is the parabola and is the envelope of its tangents.

A curve of double curvature may be considered as generated by the motion of a plane whose successive positions intersect each other in lines which constitute successive elements of the curve. For example, just previously

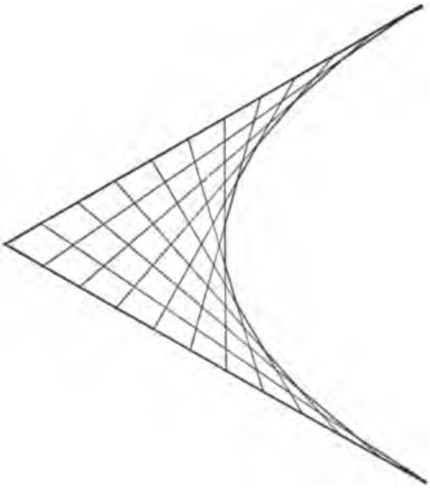

FIGURE 48.

it was stated that if no four consecutive points, or three consecutive elements connecting these points, lie in the same plane the curve is a curve of double curvature. Conceive any four such points connected by elements, one through points 1 and 2, another through 2 and 3 and a third through 3 and 4. A plane can be passed through the first and second elements, and another through the second and third, but the two planes will intersect each other in the second element, i.e., 2 and 3 and this is an element of the curve also; in like manner the successive planes will intersect each other two and two in successive elements of the curve.

To sum up the classification of lines and give illustrative examples of them, however by no means complete, the following table is appended:—

85. Table of classification of lines.

- Lines
  - Right Lines
  - Curved Lines
    - Single Curved Lines
      - Conics
        - Circle
        - Ellipse
        - Parabola
        - Hyperbola
      - Spirals
        - Archimedian
        - Logarithmic or equiangular
        - Hyperbolic or reciprocal
        - Involute of a circle, etc.
      - Trigonometric or Transcendental
        - Sinusoid
        - Tangents
        - Secants, etc.
      - Roulettes
        - Cycloid
      - Trochoids, etc.
        - Epicycloid
        - Hypocycloid, etc.
    - Double Curved Lines
      - Helix
      - Conical helix, etc.

86. Definitions.

The conics have been defined as the locii of moving points. They are also known from their properties as sections of a cone.

A circle is the section of a right circular cone made by a plane perpendicular to its axis.

An ellipse is the section of a right circular cone made by a plane at a greater angle to the axis than that made by the elements of the cone.

A parabola is the section of a right circular cone made by a plane at an angle to the axis equal to that made by the elements of the cone.

A hyperbola is the section of a right circular cone made by a plane at an angle to the axis less than that made by the elements of the cone.

* *The roulettes* are a class of curves traced by a point upon one curve when it rolls upon another curve. *The cycloid*, as generally understood, is the path traced by a point on a circle, when it rolls upon a straight line, hence belongs to the roulettes.

When both the rolling and fixed curves are circles, the paths traced are called trochoids. *The epicycloid* is the path traced by a point, on a circle which rolls upon the outside of another circle. *The hypocycloid* is the path traced by a point on a circle which rolls upon the inside of another circle.

The rolling circle is called the *generatrix* and the fixed circle the *directrix*.

The helix is defined as a curve traced by a point which moves at a uniform rate around a cylinder or cone, and at the same time at a uniform rate in a direction parallel to the axis of the cylinder or cone. It is also defined as the shortest line that can be drawn upon a cylinder or cone, between two points, which lie neither upon the same right section or upon the same right line element. *The term cylinder is not limited.* The shortest line which could be drawn in such a manner upon a cone would be known as a *conical helix.*

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* From F. N. Willson's classification in 'Theoretical and Practical Graphics.'

The projection of a conical helix upon the base of the cone, when it is a right circular cone, is the archimedian spiral mentioned at the head of the list of spirals.

87. Every surface can be generated by the motion of a line; the line may be straight or curved.

All surfaces may be classified as *ruled* or *double curved.* A *ruled surface* is generated by the motion of a straight line. A *double curved* surface is generated only by the motion of a curve.

Ruled surfaces are further divided into three classes, according to the law of motion of the generating line: *1. Planes, 2. Single curved surfaces, 3. Warped surfaces.*

A *plane* is unique and may be defined as generated by a straight line moving so that it touches another straight line and has its successive positions parallel one to another,

A *single curved surface* is defined as generated by a straight line moving so that any two but no three of its consective positions are in the same plane. These surfaces are further divided into three groupes, according to the way in which the elements intersect each other; *the cylinders*, in which a common point of intersection of the successive elements is at infinity; *the cones*, in which a common point of intersection of the successive elements is in a finite position; *the convolutes*, in which the elements intersect two and two, no three intersecting in a common point. These latter are formed by a line moving so that it is tangent to a curve of double curvature.

A notable characteristic of these surfaces, in distinction to warped surfaces and to double curved surfaces, is that they can be developed, that is they can be unrolled and laid out flat, for the plane of any two consecutive elements can be rolled upon one of the elements until it coincides with the plane formed by this element and the next consecutive element.

Warped surfaces are generated by a straight line moving so that no two of its successive positions intersect each other. The motion of the line may be controlled in several ways and an infinite variety of warped surfaces obtained. This class of surfaces will be discussed more fully a little later.*

The warped surfaces are distinguished from the single curved surfaces and are like the double curved surfaces in that they cannot be developed, for they do not consist of a succession of intersecting planes.

Double curved surfaces are generated only by the motion of curves and have no straight line elements. The most familiar double curved surfaces are those which are also surfaces of revolution, shortly to be defined, namely, the sphere, ellipsoid of revolution, paraboloid of revolution, torus, etc.

Surfaces of revolution are generated by the revolution of one line, curved or straight, about a straight line as an axis. This class does not belong to any one alone of the groups before mentioned, but it partakes of their properties. For example, *if a right line revolves about another line, to which it is parallel*, it will generate a cylinder, which is both a single curved surface and one of revolution. *If a right line revolves about another line which it does not intersect* it will generate what is known as a hyperboloid of revolution, which is both a surface of revolution and a warped surface. The latter is the only surface which has both of these characteristics. This particular surface may also be generated by the revolution of an hyperbola about its transverse axis. *If a curve is revolved about a straight line as an axis* it will, with the exception above mentioned, generate a double curved surface of revolution as the sphere, the ellipsoid of revolution, the paraboloid of revolution, hyperboloid of revolution, etc.

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*A warped surface may also be generated by the motion of a curve which changes its shape according to a certain law.

In fact surfaces of revolution may be classified under ruled surfaces and double curved. Under the ruled, come only the cylinders and cones, and the hyperboloid of revolution, the two former being single curved surfaces, the latter warped. Under double curved, come all those generated by the revolution of a curve, an infinite number, varying according to the shape of the curve.

To sum up the general classification, first, of surfaces as a whole, then of surfaces of revolution, giving illustrative examples, not complete however in all cases, the following tables are appended:

88. Table of Classification of Surfaces.

- Surfaces
  - Ruled
    - Planes
    - Single Curved
      - Cylinder
      - Cones
      - Convolutes
    - Warped, including one surface of revolution
      - Hyperbolic paraboloid
      - Right conoid
      - Oblique conoid
      - Helicoid
      - Hyperboloid of revolution, etc.
  - Double curved including surfaces of revolution
    - of Revolution
      - Sphere
      - Ellipsoids, etc.
    - Variable ellipsoid
    - Variable paraboloid, etc.

89. Table of Classification of Surfaces of Revolution.

- Surfaces of Revolution
  - Ruled
    - Single curved
      - Right circular cylinder
      - Right circular cone
    - Warped
      - Hyperboloid of revolution
  - Double curved
    - Sphere
    - Ellipsoid
    - Paraboloid
    - Hyperboloid
    - Torus, etc,

90. The method of generation of particular surfaces is not limited to that which has just been described. For illustration a cone may be generated by a right line passing through a fixed point and moving so as to touch a curve. Or again by the motion of a plane curve which changes its shape according to a certain law so that all of its points describe rectilinear paths meeting in a common point.

Although the double curved surfaces, with which we are familiar, are also surfaces of revolution, still among the infinite forms of such surfaces which are not, are several notable ones, for example the ellipsoid of unequal axes, i.e., a surface formed by the motion of a variable ellipse about one of its axes such that its points describe elliptical paths whose centers are in the axis, and also the variable paraboloid similarly formed by a variable parabola moving about its axis, each point describing an elliptical path whose center is in the axis. In these forms, sections perpendicular to the fixed line or axis are ellipses.

91. A curve is projected by projecting points in the curve. All curves whether of single or double curvature are projected as curves of single curvature because all their points lie in the coordinate planes respectively. *A curve may be indeterminate* like a straight line, when its plane is perpendicular to both coordinate planes. *If a curve is projected upon a plane, to which it is parallel*, it will be a parallel, and equal curve. *The traces of a curve* are determined in the same manner as those of a straight line.

92. THEOREM IX.

Two projections of a curve being given, the curve will, in general, be completely determined.

Proof:—For, if at the horizontal projection of any point

on the curve, a perpendicular is erected, to the H plane it will contain a point of the curve and similarly if at the vertical projection of the same point, a perpendicular is erected to the V plane it will contain the same point of the curve, hence the point, being common to two lines, must lie at their intersection, and so for any other point of the curve.

93. One line is tangent to another when it has at least two consecutive points in common with it.

Assume a secant to any curve, which cuts it in two points, to be moved, by rotation or translation, until these two points of intersection approach one another and become coincident points; the secant approaches tangency and become such when the two points become coincident.

A secant may, in some cases, be moved until more than two points become coincident. This may be illustrated by a secant which cuts a curve, having a point of inflection, between the points common to it and the secant. Further, a straight line, may be tangent to a curve at two or more consecutive points, and also tangent elsewhere at two or more other consecutive points or cut the curve again in one or more points.

In analystic geometry, the degree of the equation to any curve, is an index of the maximum number of points in which a straight line may cut the curve. Of course, some points may be imaginary.

If a curve and a straight line continually approach one another, but do not meet, they have a point in common; i.e., are tangent at infinity. In this case, the straight line is said to be an asymptote to the curve or they are asymptotic to each other.

Two curves are tangent to each other when they have at least two points in common or, at a pair of common consecutive points, they have a common tangent.

If a right line is tangent to a curve of single curvature, it will be contained within the plane of the curve, for by definition of a curve of single curvature, if two consecutive points be connected by a straight line, these points lying in the plane, the straight line connecting them lies wholly in the plane.

If a right line is tangent to another right line it coincides with it.

94. THEOREM X.

If two lines are tangent in space their projections on the same plane are tangent to each other.

> **Proof:—For the two or more consecutive points in which the lines are tangent to each other are projected as consecutive points of the projection of both lines, hence fulfill the conditions of a tangent to both lines.**

The converse of this is not true unless the consecutive points in projection are the projections of the same points of each line.

Although analytically a tangent touches a curve in two or more points, it is customary to speak of the 'point of tangency.'

95. A normal to a curve is a line normal to the tangent to the curve at the point of tangency.

An infinite number of perpendiculars can be drawn to a line at a given point constituting a perpendicular plane at the point, hence a plane perpendicular to a tangent to a curve at the point of tangency will contain the normals which can be drawn to the curve at that particular point. But the normal to a curve of single curvature is generally understood to mean that normal which lies in the plane of the tangent and the curve, and will be so spoken of in this book.

If a curve of single curvature is rolled upon its tangent until a given linear distance upon it comes successively into contact with the tangent, the curve is said to be rectified.

96. To draw a tangent to an irregular plane curve at a point on the curve.

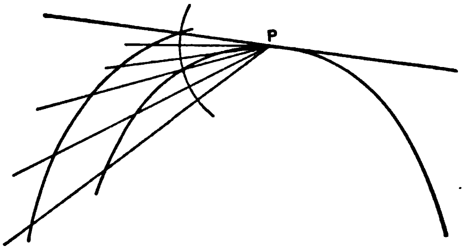

FIGURE 49.

Let the curve be given as in Fig. 49, and the point of tangency P. Draw a series of chords to the curve through P, then with P as a center and any radius draw an arc of an auxiliary circle to cut these cords. Lay off the length of each chord, (i.e., the distance from P to the curve) upon the chord produced and measuring from, and on the same side of, the auxiliary circle that the chord lies with respect to the point P. Through these points draw a smooth curve. Where it intersects the auxiliary circle will be a point in the tangent.

97. To find the approximate point of tangency of an irregular plane curve and its tangent.

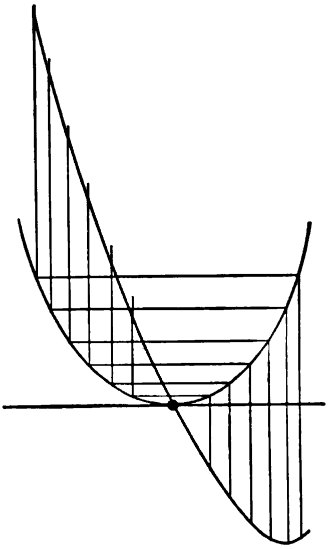

FIGURE 50.

Let the curve and its tangent be given as in Fig. 50. Draw a series of chords of the curve parallel to the given tangent. At the intersection of these chords, respectively, with the curve, erect perpendiculars to the chords and lay off upon them, respectively, distances measured in opposite directions equal in length to the chords terminating in the perpendiculars. Draw a smooth curve through the points located and it will intersect the given curve in the point of tangency.

The two preceding constructions are now and then useful, but ordinarily a tangent to a curve can be drawn with all necessary accuracy, in the first example, by placing a straight edge to touch the point and the curve, and in the second example the point of tangency can be estimated well enough for all practical purposes, in those cases where the curve does not closely approach the straight line tangent in direction.

98. A tangent plane to a surface is the limit approached by a secant plane when three points of the curve of intersection of the plane with the surface, which do not lie in the same straight line, become coincident.

A tangent plane may otherwise be defined as such that, if any other and secant plane be passed to cut the surface, it will cut a line from the surface and a line from the tangent plane which are tangent to each other, hence the tangent plane at a point is the locus of the tangents to plane curves which may be drawn on the surface through the point.

For this we may derive a general rule for passing a plane tangent to a surface at a given point.

Rule:—*Through the given point conceive two secant planes to be passed to cut the surface in at least two plane curves. Draw tangents to these at the given point and the plane of the tangents will be the plane required.*

A right line is normal to a surface, at a given point, when it is perpendicular to the tangent plane at that point.

A plane is normal to a surface at a given point, when it contains a line normal to the surface at the point. Hence, while there can be but one normal line, there can be an infinite number of normal planes at a given point on a surface.

CHAPTER V.

SINGLE CURVED SURFACES.

99. **THEOREM XI.**

A plane which contains two consecutive rectilinear elements of a single curved surface will be tangent to the surface throughout these elements,* and conversely, if a plane is tangent to a single curved surface, it will contain at least two consecutive rectilinear elements of the surface.

> **Proof:—For, if through any point of one element, a secant plane is passed, it will cut a curve from the surface having consecutive points common to both elements. A line connecting these points will (by Sec. 93) lie in the plane of the curve of the section and (by Sec. 98) in the tangent plane. Hence the given plane will be tangent at the assumed point or all along the element which is the locus of the assumed points.**

100. **A cylinder is defined** as a single curved surface generated by a line which moves so that its successive positions are parallel, and touch a curve. In the most common interpretation, the curve is a plane curve.

***While the words, 'these elements' are used, it is understood that the common conception of 'an element of tangency' prevails.**

The curve of intersection with a cylinder, of a plane perpendicular to the generatrix or to the axis of the cylinder, is that which gives the cylinder it characteristic name as elliptical, hyperbolic, parabolic cylinder, etc.*

A cylinder may also be called an oblique cylinder, when considered merely in reference to a section oblique to the generatrix, and then the name of the cylinder is commonly derived from the shape of the oblique section.

Any secant plane of a cylinder, making a greater angle than zero degrees with the elements, may be considered as a base of the cylinder and may also give it its characteristic name. It is customary to call the curves of section, of two parallel secant planes, 'the bases' of the cylinder.

A right cylinder is one whose base or bases are perpendicular to the axis or elements.

A right cylinder with a circular base is also called a cylinder of revolution since it may be generated by one line revolving about another to which it is parallel.

Let Fig. 51, page 100, be a cylinder with circular base in H and another base parallel to H with elements oblique to the plane of the base. This cylinder is called an elliptical cylinder according to the generic definition just given, since a section perpendicular to the axis would be an ellipse. It may be called an oblique circular cylinder, when referred to its oblique bases.

101. To assume a point on any single curved surface, assume one projection and draw the projection of an element through the point. The points in which this element

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* The axis goes through the center, focus or other characteristic point of the curve and is parallel to the generatrix. Such a point may be a point of changing curvature of the curve of tangents, the cusp in a cuspidal curve, etc.

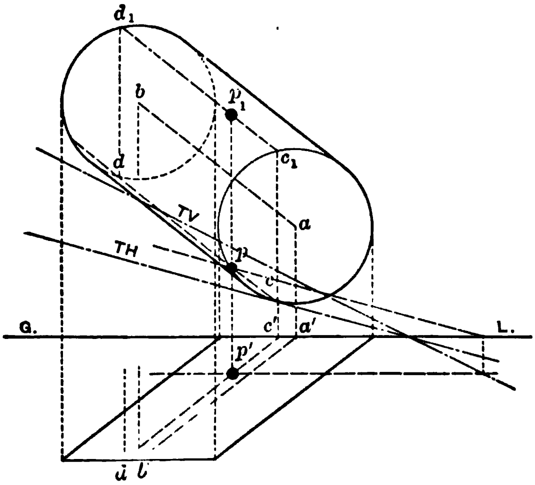

FIGURE 51.

cuts the curve or curves of the bases give points in the corresponding projection of the element; and the corresponding projection, of the required point lies on the corresponding projection of the element through the point. An element of a single curved surface may be assumed in like manner.

For example, let p' be assumed on the cylinder in Fig. 51. Draw the V projection of an element CD through the point p'. Get the H projection of this element, and through p' erect a perpendicular to the G.L. to cut the H projection of the element CD in the point p which is the corresponding projection of the point required.

Note that p' would stand for two points upon the surface of the cylinder, for through the point P a perpendicular can be drawn to the V plane and it would pierce the surface in two points, p_1 being the other H projection of the point corresponding to p'.

102. To draw a tangent plane to a cylinder through any point on the surface.

Analysis:—*The required plane will contain the tangents to curves of section of the surface, but an element of contact may be taken as one line in the tangent plane, a tangent to the curve of intersection of the surface, with a coordinate plane, may be taken as the other line, but the latter is a trace of the plane since it lies in the coordinate plane. Hence, draw an element of the surface through the given point and where this element cuts the coordinate plane, in the curve of the base, draw a tangent to the base. The plane of these two lines is the required plane.*

If the plane of the base does not lie in a coordinate plane, draw a tangent to the curve of the base at the point of intersection, with it, of the element of contact. This, and the element of contact, are two lines of the required plane. The traces of the latter go through the corresponding traces of these lines.

Construction—Let the cylinder be given as in Fig. 51 and the given point P. Draw the element DC through P and at *c*, the H trace of DC, draw a tangent to the curve of the base. It is the H trace of the required plane. With the H trace and P as a point in the plane, find the V trace by means of a horizontal of the plane.

If the axis of the cylinder is perpendicular to one coordinate plane, the trace of the tangent plane upon the corresponding plane of projection would be perpendicular to the G.L.

If the axis of the cylinder is parallel and its bases perpendicular to both coordinate planes, it is necessary to use an end projection to get the points of intersection of the elements with the curves of the bases.

103. To pass a plane tangent to a cylinder through a point outside.

Analysis:—*If an auxiliary line is drawn through the given point, parallel to the elements of the cylinder, this line will lie in the tangent plane through the point. The plane of this line and the element of contact of the tangent plane will be the required plane. But the secant plane of the base will cut the tangent plane in a line, which is tangent to the curve of the base, and the secant plane intersects the auxiliary line drawn through the given point parallel to the elements of the surface. Therefore, find the intersection ot the auxiliary line with the plane of the base, and draw through this point a line tangent to the curve of the base. The plane of these two lines is the required plane. There can, in general, be two such planes as two tangents can, in general, be drawn to the curve of the base from a given point outside.*

Construction:—Let the cylinder be given as in Fig. 52 in which the plane of a base is in V and its center in the G.L. Let P be the given point.

Draw an auxiliary line PQ through the point P intersecting the plane of the base in Q. Q is one point in a tangent to the curve of the base, the point of tangency being C. QC is a line of the required plane and since it lies in V also, is the V trace of the required plane T. CD, an element of the cylinder through C, is another line of the tangent plane. Using the V trace and P, a point in the plane, a vertical of the plane locates a point in the H trace.

In this problem also a second tangent plane R may be drawn.

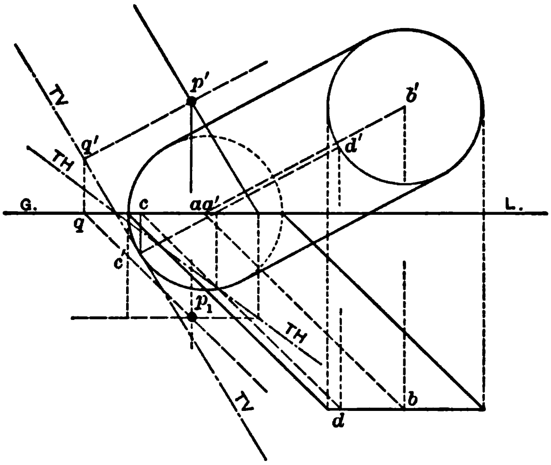

FIGURE 52.

104. To draw a tangent plane to a cylinder and parallel to a line outside the cylinder.

Analysis:—*The required tangent plane must contain a line parallel to the given line, and similarly, if a plane is passed through the given line and parallel to the elements of the cylinder it will be parallel to the required plane. Hence draw a line parallel to the elements of the cylinder to intersect the given line. The plane of these two lines is parallel to the required plane.*

To draw a parallel plane tangent to the cylinder, produce the elements of the cylinder, if necessary, to intersect a coordinate plane, giving the curve of intersection with the latter. Tangent to this curve and parallel to the similar trace of the auxiliary plane, draw the trace of the required plane, from which the corresponding trace may be obtained.

If the axis of the cylinder is parallel to the G.L. and the bases perpendicular to it, an end plane will have to be used to get the traces of the auxiliary and the parallel tangent planes.

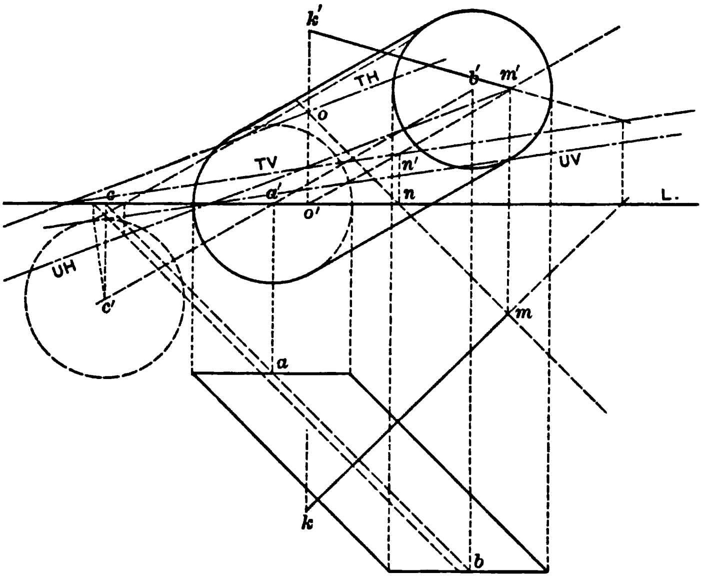

FIGURE 53.

Construction:—Let the cylinder be given as in Figure 53, with bases parallel to V and centers of bases at A and B respectively; and let the given line be KM. Through any point of KM, as M, draw a line MNO, parallel to the elements of the cylinder. Through its traces, and those of the line KM, draw the traces of their plane T. Prolong the elements of the cylinder and the axis to pierce the V plane, the latter in the point C. Since the base, with center at A, is parallel to V, the intersection of the cylinder with V will be also a circle, with center at C. Tangent to the circle of the base in V, draw the V trace of a tangent plane U parallel to T. By means of the H trace of the element of contact of this tangent plane, draw the H trace of the plane U parallel to that of the plane T. Two tangent planes can be drawn to fulfill the conditions.

A plane cannot be drawn tangent to a cylinder, and to contain a given line outside, unless that line, prolonged, is tangent to the surface at a finite or infinitely distant point.

In locating the traces of tangent planes, and in finding the second trace when one trace has been found, the several methods may be used which have already been studied in the fundamental problems. (1.) A point in the plane can be taken, and a horizontal, a vertical, or any other auxiliary line drawn through the point to lie in the plane. (2.) Any line, known to be in the required plane, can, by its traces, locate the trace of the desired plane.

If the traces of the required plane do not intersect the G. L. within the limits of the drawing, it will be generally necessary to find the traces of two lines in the plane, unless the required plane is parallel to some other given or derived plane. Otherwise, the trace of only one auxiliary line need be obtained.

If the axis of the cylinder is parallel to a coordinate plane, the trace of a tangent plane, upon that coordinate plane, will be parallel to that projection of the elements of the cylinder.

105. To pass a plane normal to a cylinder through a point on the surface.

Analysis:—*The normal plane will be perpendicular to the tangent plane at the point, i.e., contain a perpendicular to the tangent plane through the given point. Hence find the tangent plane, (by Section 102), and, through the* *latter, draw a perpendicular to this plane. Any plane, to contain this line, will satisfy the conditions.*

106. To pass a plane normal to a cylinder through a point outside the cylinder.

Analysis:—*The normal plane will be perpendicular to the tangent plane and contain the point, i.e., contain a perpendicular to the tangent plane and passing through the point. Hence, find the tangent plane, to the cylinder, through the given point, (by Section 103), and then draw a perpendicular to this plane, through the point. Any plane passed through this perpendicular, will satisfy the conditions*

107. To pass a plane normal to a cylinder and parallel to a given line.

Analysis:—*The normal plane will be perpendicular to the tangent plane, which, in turn, is parallel to the given line. Hence, find the tangent plane to the cylinder, parallel to the given line, (by Section 104.) At any point of the tangent plane, erect a perpendicular, and, through, any point of this perpendicular, draw a line parallel to the given line. The plane of these two lines is the required plane.*

108. To draw a plane normal to a cylinder, through a given point on the cylinder, and parallel to a given line.

Analysis:—*The normal plane will be perpendicular to that tangent plane which, in turn, is parallel to the given line. Hence, find the tangent plane to the cylinder which is parallel to the given line. Through the given point on the cylinder, draw a perpendicular to this tangent plane, then through any point on this perpendicular, draw a line* *parallel to the given line. The plane of these two lines is the required plane.*

109. The elements of contour of a solid are determined by the lines of tangency of planes respectively perpendicular to the coordinate planes, and these planes go to make up the projecting cylinder of the solid, on the respective coordinate planes. In the case of cylinders and cones, the contour is partly made up of the projections of elements.

It can be seen that the contour elements of the vertical projection, will not be identical with those of the H projection and, therefore, each set of elements of contour has its corresponding projections which do not coincide with that of the other.

110. The curve of intersection of a plane and a surface is found by plotting the points of intersection of lines, lying respectively in each. And, for this purpose, the following general rule is given to cover all kinds of surfaces:

Rule:—*To find the curve of intersection of a plane and a surface, pass a series of auxiliary secant planes, so chosen as to cut straight line elements, circular section or such lines from the solid as may easily be drawn in projection; these secant planes will cut right lines from the intersecting plane surface, the intersections, of which, with the elements or lines cut from the surface by the secant planes, will give points in the curve of the section.*

In single curved surfaces, and warped surfaces, it is always posssible to cut out straight line elements, although these may not in all cases be the most convenient kind of lines to cut.

Rule:—*To find the piercing point of a line with a surface, pass any convenient plane through the line to cut the surface, Find the curve of intersection of this plane and the surface. The required piercing point will lie at the intersection of the given line and the curve cut from the surface.* The line may pierce the surface in more than one point.

111. The development of a surface of single curvature is the area of contact of the surface, when it is rolled at one of its elements upon a tangent plane to that element until the planes of the consective elements come successively into contact with the tangent plane. Only surfaces of single curvature and polyhedrons can be developed, in the true sense of the word, although any surface is capable of a more or less close approximation to development, when it has to be constructed in practical work. An entire surface may be developed, or any portion of it, lying between two chosen elements and any other limiting lines.

112. To find the intersection of a right cylinder with a plane surface oblique to its axis and to draw a tangent to the curve of intersection at any point.

Analysis:—*The intersection is a plane curve, intersecting all the elements of the cylinder. Hence, pass secant planes to cut straight lines (elements) from the cylinder, and also straight lines from the plane, the points of intersection, respectively, of which give points in the curve of the section.*

The tangent to the curve of intersection, at any point by Theorem X, Sec. 94, will be projected as a tangent to the projection of the curve, but it is most accurately drawn by revolving the curve of intersection about the trace of its plane into the coordinate plane where, by the properties of the curve, it may be more accurately drawn, and then revolved back again, noting that the trace of the tangent is a fixed point in the revolution.

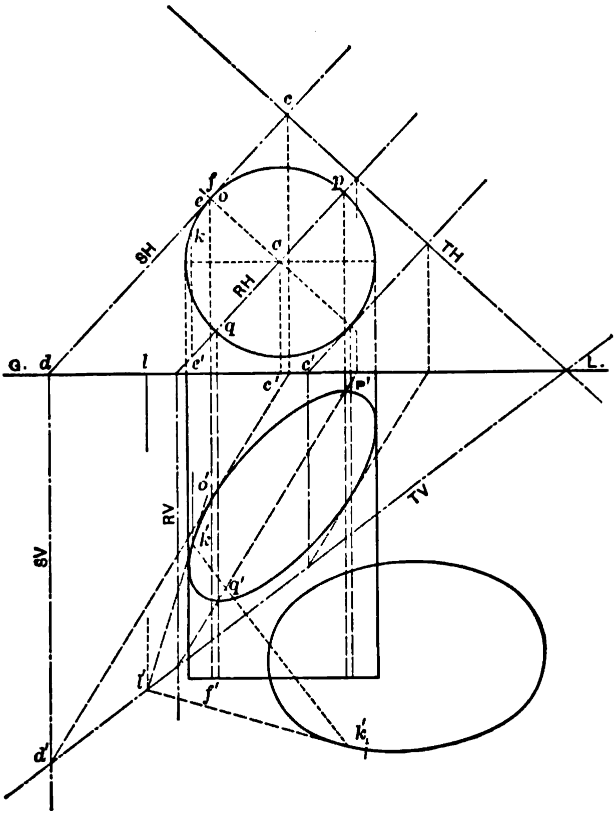

FIGURE 54.

Construction:—Let the cylinder be a right circular cylinder, with base in H, as shown in Figure 54. The center of the base is at A and let the plane be given as T. Auxiliary planes, to cut straight line elements from the cylinder, are parallel to the axis of the cylinder, in this case perpendicular to H. Let S be such a plane, cutting T in the line CD, and tangent to the cylinder along the element EF. The point of intersection, O, of the line CD and the element cut from the cylinder, is a point in the curve of intersection of the plane T and the cylinder. In a similar manner, an auxiliary plane R, intersecting T in a line parallel to that made by the plane S, gives P and Q, two other points in the curve of intersection. And so on, for as many points desired.

Let K be a chosen point, on the curve of intersection, at which to draw a tangent. Revolve the curve of intersection into V, about the V trace, where the tangent can be drawn more accurately, k', l' is the revolved position of the tangent. The point L, in which the tangent cuts the V trace, is a fixed point in the revolution. Revolve the tangent back again into true position in the plane T.

Auxiliary planes could have been passed in any chosen direction, for example, radially through the axis of the cylinder, or parallel to each other in any other direction. In this particular case, auxiliary planes, parallel to H, might quite as conveniently be chosen; they would have intersected the cylinder in circles, readily drawn, and their intersections found, with the horizontals cut from the plane T.

113. To develop, approximately, the right circular cylinder of Sec. 112, and to trace on it the curve of intersection with the plane T.

Analysis:—*From definition (Section 111) the develop-*

ment of a right cylinder with parallel bases, is bounded by the rectified curves of the bases and the elements perpendicular to and connecting the extremities of the rectified bases.

Hence, choose any element, and, perpendicular to it at its extremities, lay off the rectified curves of the bases, in any convenient manner, i.e., for example, using short chord lengths as equal to the arcs which subtend them. Connect the last divisions on the rectified curves to constitute the element consecutive with the first one assumed. The curve of intersection of a secant plane will be a curve connecting the proper points of the respective elements as found in the development. Hence, lay off on each intersected element in the development, a distance from the rectified curve of a base that the point of intersection of the element with the secant plane is from that base; draw a smooth curve through these points.

Construction:—Take any reference line as the upper one in Figure 55, and lay off on it successively, starting at the left, and going toward the right, the chord lengths

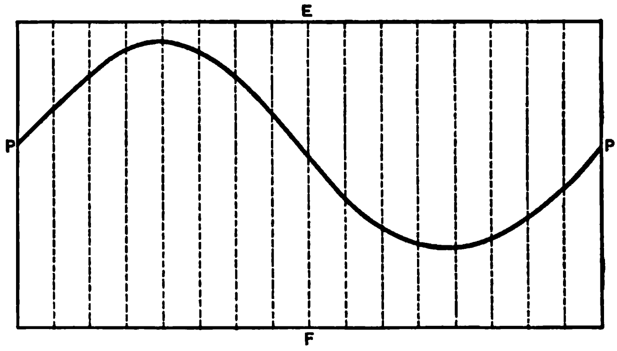

FIGURE 55.

into which the base may have been divided by the auxiliary planes in H projection, beginning at *p* and proceeding counter clockwise around the base. At these divisions erect perpendiculars to the reference line or base, and, making them as long as the elements are long, obtain the other base. Next, divide the elements of the development into the same proportionate parts that the elements are divided in projection by the secant plane. A smooth curve connecting these points will give the developed curve of intersection.

114. To develop an oblique cylinder.

Analysis:—*To develop any cylinder, it is desirable to obtain, first, a right section. Then rectify this, and erect the elements perpendicular to it; lay off on these the respective lengths between the bases measured from the right section as a reference line.*

If the axis of the cylinder is oblique to both coordinate planes, it is most convenient to change the latter so that the axis is parallel to one of them, for then, one projection of the curve of intersection will be a straight line, and the construction for getting the other projection of the intersection is very much simplified.

Construction:—Let an oblique elliptical cylinder with parallel bases, and axis AB oblique to H and V, be given as in Figure 56. Assume any plane T, conveniently between the bases of the cylinder to be perpendicular to the axis. Its traces are respectively perpendicular to the projections of the axis of the cylinder. To find the curve of intersection of T with the cylinder, project the latter upon a new V_1 plane parallel to the axis cutting H in $G_1 . L_1$. The new

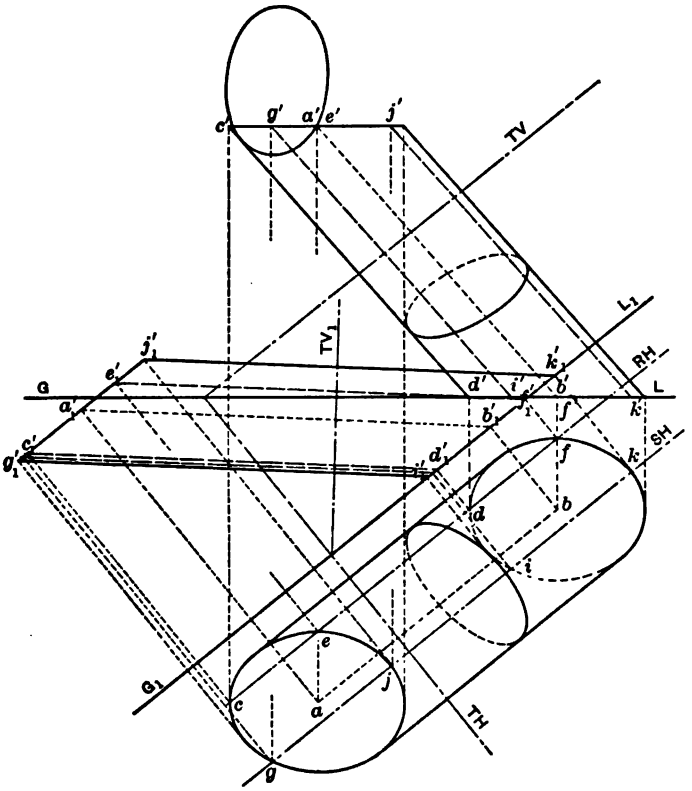

FIGURE 56.

V_1 trace of T will be perpendicular to the V_1 projection of the axis $a'_1\ b'_1$. Pass planes perpendicular to H and to T, as R and S for example, to cut elements CD, EF and GI, JK respectively from the cylinder as shown. The V_1 projections of the piercing points of the elements lie in the V_1 trace of T; the H projections lie at the intersection of perpendiculars to the $G_1.L_1$. through these points and the H projections of the respective elements cut from the cylinder.

To get the true curve of the intersection, revolve it into or parallel to the V plane, for example about the V trace of T as shown above TV on the figure.

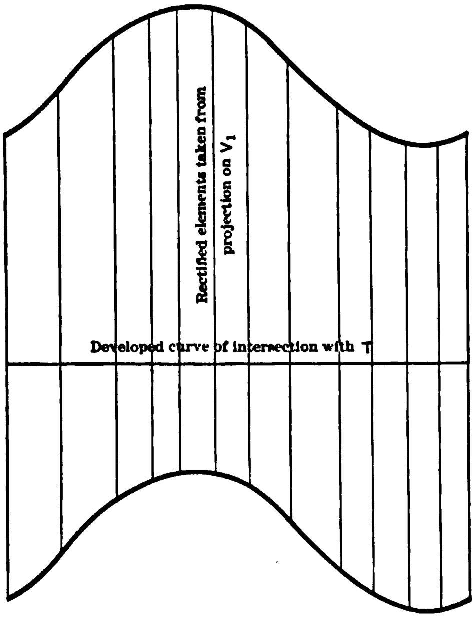
Rectified elements taken from
projection on V_1
Developed curve of intersection with T

FIGURE 57.

Next, rectify the curve of the section on any base or reference line, as in Fig. 57, by laying off on it any convenient chord lengths as taken from the rectified curve of section and erecting perpendiculars to the points of division. Lay off on these perpendiculars, either side of the rectified curve of the section, distances respectively equal to the segments of the elements upon either side of the curve of section. The true lengths of all elements are projected upon the V_1 plane and are obtained directly.

115. A cone is defined as a single curved surface generated by the motion of a line which goes through a fixed point and touches a curve. There is nothing which limits the curve but it is usual to consider it a plane curve.

The generic or characteristic name of a cone is derived from the perimeter of a plane section which is so taken that a line through the apex of the cone, and perpendicular to the plane section, goes through its center, focus, or other characteristic point of the curve.* Some cones so defined are elliptical, hyperbolic, parabolic, etc. The axis of a cone is a line which goes through the apex and pierces a plane section, arbitrarily chosen as the base, in its center, focus, or other characteristic point. The axis of a generic, or right cone, is such a line which is, further, perpendicular to the plane of the base.

A cone may be at the same time a right cone and an oblique cone. The definition just given for a generic cone makes every cone some form of right cone. Any plane section, however, oblique to the generic axis may also give the cone a name. A simple illustration will suffice: A right circular cone, if cut by a plane oblique to its axis, may also be called an oblique elliptical cone. A cone is generally defined in terms of the base section whether it be oblique to the axis or perpendicular to it.

A cone of revolution is a right circular cone, because it can be generated by the revolution of a line about another line which it intersects.

The generating line or generatrix of any cone is not limited by the apex of the cone, therefore, a line always generates two parts, each on opposite sides of the fixed point. These parts are called *nappes*, as the right and left nappe respectively, or the upper and lower nappe.

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*The author believes this to be an uncommon definition and therefore requires some fuller explanation. The 'characteristic point' may mean a point of inflection of the curve of section if it has sinuous like characteristics, a cusp, or a vertex, etc.

116. To assume a point on a cone, assume either projection of the point, and through it, draw the projection of an element, which is a line of the surface passing through the apex. Note where it cuts the curve of the base, with this point construct the other projection of the element. The other projection of the assumed point will be on the corresponding projection of the element. Each assumed point lies, in general, on the projection of two elements. Either one can be taken at will. An element of the cone may be assumed in a similar manner.

117. To draw a tangent plane to a cone at any point on the surface.

Analysis:—*The plane will contain the tangent to two curves of section of the surface through the point, but an element of contact, may be taken as one line of the tangent plane, a tangent to the curve of intersection of the surface with a coordinate plane may be taken as the other line, but the latter is also a trace of the plane, since it lies in the coordinate plane. Hence, draw an element of the surface through the given point and where this cuts the coordinate plane in the curve of a base in that plane, draw a tangent to the base. The plane of these two lines is the required plane.*

If the plane of the base does not lie in a coordinate plane, draw a tangent to the curve of the base at the point of intersection with it of the element of contact. This and *the element of contact are lines of the required plane. The traces of the tangent plane go through the respective traces of these lines.*

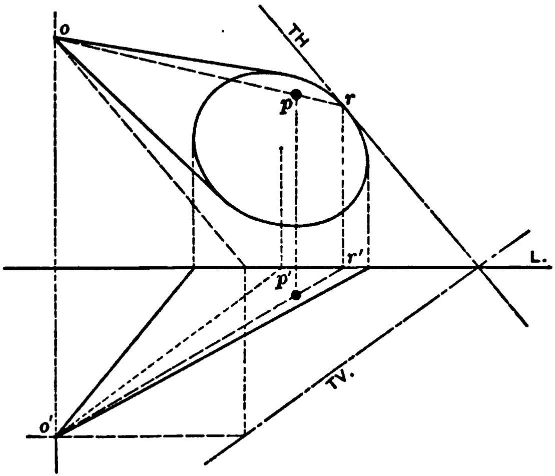

FIGURE 58.

Construction:—Let an oblique elliptical cone be given as in figure 58 with its elliptical base in H and apex at O, and the point P by its horizontal projection. Find *p'* by drawing the element *o p* to cut the curve of the base in *r*. *r'* is in the G. L. and *p'* is on *r'o'*. The required tangent plane contains the element O P R and since the base of the cone is in H, a tangent to the curve of the base at *r*, the foot of the element, is the H trace of the required plane. The V trace is obtained by using a horizontal of the plane through O as shown.

118. To pass a plane tangent to a cone through a point outside.

Analysis:—*If an auxiliary line is drawn through the given point and the apex of the cone, it will be a line of the*

tangent plane, for all tangent planes to the cone go through the apex. Another line of the tangent plane is a line lying in any secant plane, intersecting the auxiliary line mentioned and tangent to the curve of intersection with the cone of the secant plane. If the base of the cone is in a coordinate plane, this second line would be that passing through the trace of the auxiliary line and tangent to the curve of the base in the coordinate plane; it would furthermore be a trace of the tangent plane. The other trace may be obtained by any of the usual methods.

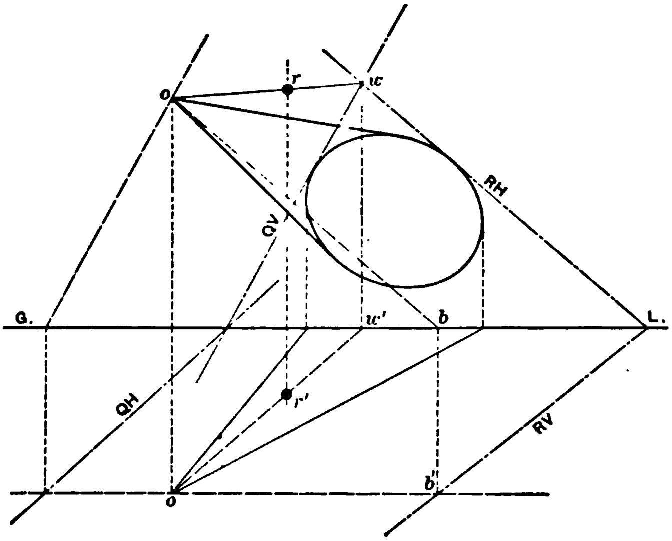

FIGURE 59

Construction:—Let an oblique elliptical cone be given as in Fig. 59 with base in H and apex at O as shown. Let the point outside be R. Draw a line O R through the apex of the cone to pierce H, the plane of the base in W. Through W draw a tangent to the curve of the base. This will be at once a line of the required plane R and the H trace of the plane. By means of a horizontal OB of the required plane the V trace is obtained.

By similar construction another tangent plane Q may be found to fulfill the conditions.

119. To draw a plane tangent to a cone and parallel to a line outside.

Analysis:—*The required tangent plane must contain a line parallel to the given line. Such a line can be drawn through the apex of the cone. A plane tangent to the cone and containing this line would fulfill the conditions. Hence, draw a line through the apex parallel to the given line, and through its piercing point with the plane of the base, draw a tangent to the curve of the base. This line, and the one through the apex, are lines of the required plane. Since two or more tangents can be drawn from a point to a curve, there can be two or more tangent planes.*

The construction is left to the student.

A plane cannot be drawn tangent to a cone and to contain a given line outside unless the line, prolonged if necessary, would be tangent to its surface.

If the axis of the cone is parallel to the G. L., the plane of the base will be an end plane and the construction requires to be modified to suit.

120.* To pass a plane normal to a cone through a point on the surface.

Analysis:—*The normal plane will be perepndicular to the tangent plane at the point, i.e., contain a perpendicular*

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*The analysis of the following three problems are the same as the corresponding ones for the cylinder.

to the tangent plane through the given point. Hence, find the tangent plane at the point, (by Section 117), and through the latter draw a perpendicular to this plane. Any plane to contain this line will satisfy the conditions.

121. To pass a plane normal to a cone through a point outside the cone.

Analysis:—*The normal plane will be perpendicular to the tangent plane and contain the point, i.e., contain a perpendicular to the tangent plane and passing through the point. Hence, find the tangent plane, (by Section 118), and draw a perpendicular to the plane through the point. Any plane passed through this perpendicular, will satisfy the conditions.*

122. To pass a plane normal to a cone and parallel to a given line.

Analysis:—*The normal plane will be perpendicular to the tangent plane which latter is also parallel to the given line. Hence, find the tangent plane to the cone and parallel to the given line, (by section 119.) At any point of the tangent plane erect a perpendicular, and through any point of this perpendicular draw a line parallel to the given line. The plane of these two lines is the required plane.*

123. To find the curve of intersection of any cone with a plane oblique to the axis.

Analysis:—*The intersection will be a plane curve, intersecting all the elements of the cone. Hence, pass auxiliary secant planes to cut straight lines, elements, from the cone and lines from the given plane; the points of intersection, respectively, give points in the curve of the section.*

Construction:—Let an oblique elliptical cone be given as in Figure 60, with elliptical base in H, and cut by a plane T. Pass a series of secant planes, one of which is S, through the apex of the cone, assuming at will the H traces of these planes, as SH. The V traces can be found by means of horizontals, as shown in the case of plane S. Find the lines of intersection, respectively, of these

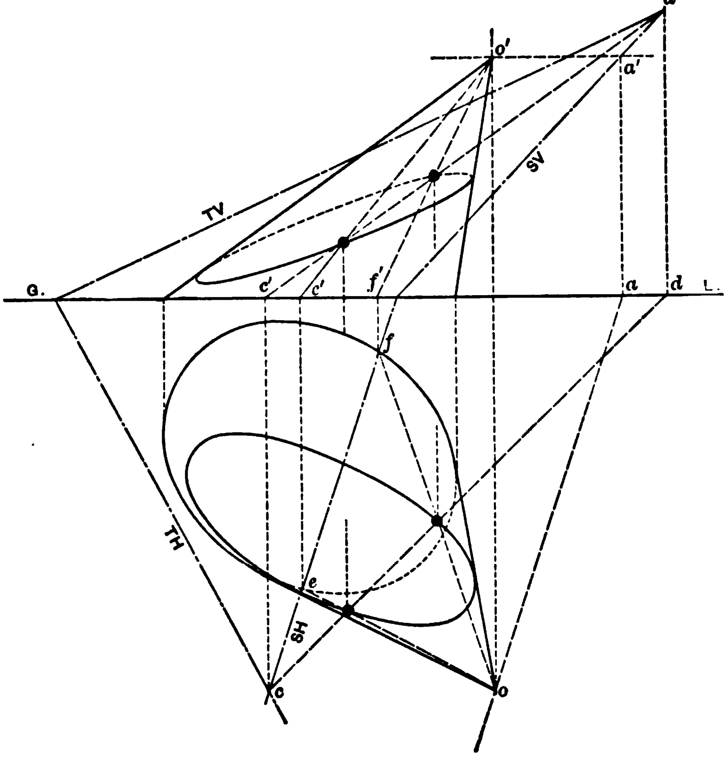

FIGURE 60.

auxiliary secant planes with the plane T, for example, CD is the line of intersection of the plane S with the plane

T. The elements cut from the cone by the plane S, are EO and FO, respectively. Where $e'o'$ cuts $c'd'$ is a point in the curve of intersection, and, where *co* cuts. *cd*, is its corresponding projection. Similarly, where $f'o'$ cuts $c'd'$, is another point in the curve of intersection and where *fo* cuts *cd*, is its corresponding projection, and so on for as many points as desired.

Either the V or the H traces of the auxiliary secant planes can be assumed in this problem in any direction which seems convenient, the corresponding traces to be determined from them.

The well known conics are derived by the intersection of a plane and a right circular cone, or cone of revolution as was discussed in Sections 86 and 115.

124. To develop a right circular cone approximately.*

Analysis:—*In a right circular cone, all the elements are of equal length. Hence, since the apex is a point common to all the elements, the development will consist of the sector of a circle, the distance between the sides of which, is determined by the length of the rectified curve of the base subtending the angle between the sides and laid out with the apex as center.*

Therefore, with a radius equal to the true length of an element, draw the arc of a circle and lay off on it from any convenient point, successively, chord lengths taken equal to short arc lengths into which the base may be divided. Connect the first and last point of the chords with the center or apex and this will be the approximate development.

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*Since the circle of the base cannot, by geometry, be accurately rectified.

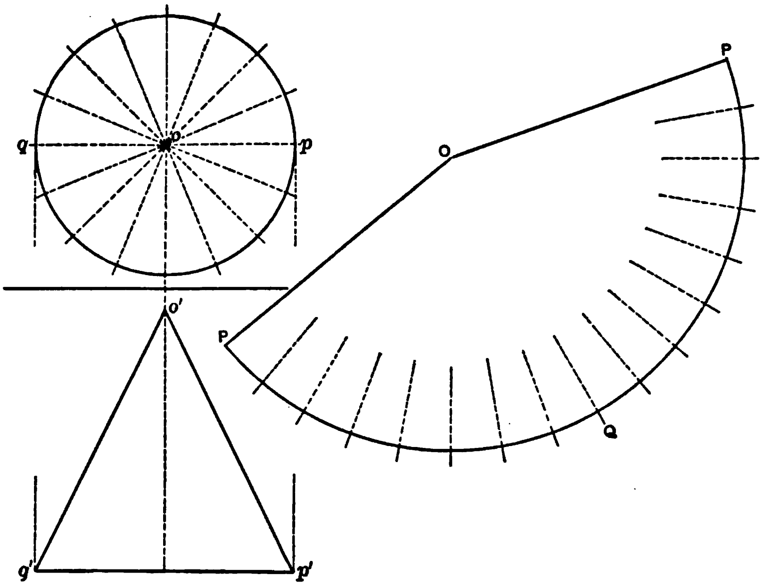

FIGURE 61

Construction.—Let a right circular cone be given, as in Figure 61, with base parallel to H and apex in O. Divide the curve of the base, for convenience, into a number of equal divisions, taking the chords, connecting these, as equal to the respective arc lengths. Choose a center O and, with a radius equal to the length of the elements of the cone, which is shown in true projection by those elements which are parallel to V as OP and OQ, describe an arc of a circle PQP and, on this circle, lay off successively the chord lengths into which the circle of the plan had been divided. Connect the first and last division with the center O and this is the approximate development.

Any other than a right circular cone will have to be developed by a different method than the foregoing.

125. The development of any cone in general, may be effected as follows:

Analysis:—*Divide the curve of the base, or intersection of the cone with a plane, into a large number of parts taking the chord lengths as equal to the arcs subtending them. Draw the corresponding elements of the cone through these divisions. Then, taking an apex or center anywhere, assume a line through it, equal in length to an element of the cone, next, with the apex as center, draw an arc of a circle of radius, equal to the true length of the next adjoining element, and with center at the extremity of the first element, and radius equal to the true chord length between the two elements, describe an arc of a circle to cut the first arc in a point which will lie upon the developed curve of the base. With centers, respectively, as before, at the apex and the extremity of the last located element, and radii equal, respectively, to the true length of the next element, and the true chord length between the elements, find the next point in the developed curve of the base and so on until all the points of the base selected have been located. The first and last elements, and a smooth curve through the points plotted, constitute the development.*

If the right section of an oblique cone is a circle then the following special method for development may also be employed:—

Analysis:—*Pass a plane perpendicular to the axis. It will cut the cone in a circle. Develop this right circular cone, by Section 124, and find points in the curve of the desired base by laying off upon the developed elements, extended if necessary, the respective rectified intercepts on those elements between the base and the curve of intersection* *with the auxiliary plane. A smooth curve through these points will give the curve of the base.*

126. To develop an oblique cone.

Analysis:—*The analysis is the same as that given for the general case at the beginning of Section 125.*

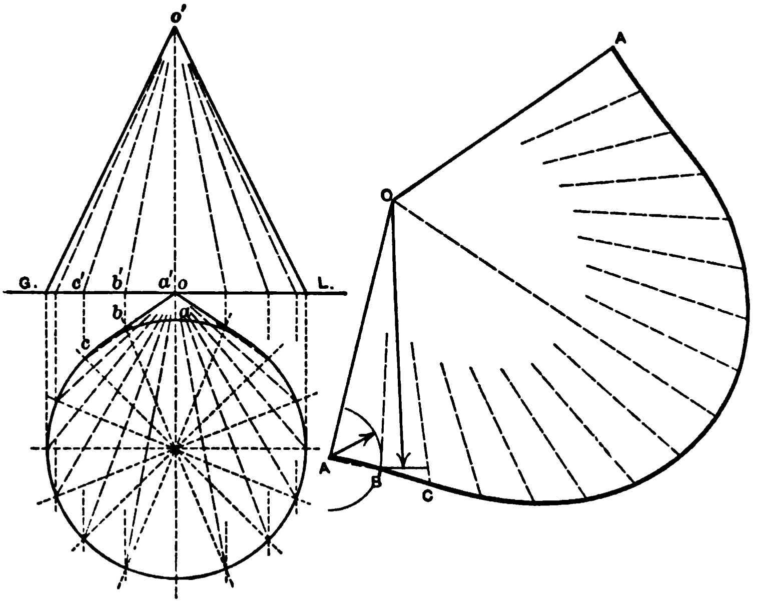

FIGURE 62.

Construction—Let an oblique cone with circular base in H and axis in an end plane be given as in Figure 62. Divide the base in H into, say, sixteen equal parts. For symmetry in the development choose element OA, to lay off in a convenient place, obtaining its true length, by revolving the element parallel to V. Find the true lengths of all the elements by similiar revolution. Next, with O, of the development, as center, and radius equal to OB, describe an arc of a circle, then, with *ab* as a radius, and center at A of the development, describe an intersecting arc obtaining the point B. Similiarly, obtain all the desired points in the curve of the base. The elements OA of the development are theoretically consecutive elements of the curve.

127. The convolutes have already been defined broadly in Sec. 87 but to repeat for the sake of clearness, they are a class of surfaces generated by the motion of a line which is tangent to a curve of double curvature.

Bearing in mind the discussion in Sec. 87, that when a line moves tangent to a curve of double curvature the elements of the surface generated intersect each other two and two, that is, two elements intersect each other in a point of the curve and pass through two other consecutive points, no three elements passing through four consecutive points, and it can be seen that the surface is developable, like other single curved surfaces.

There can be an infinite variety of convolute surfaces depending upon the form of the curve of double curvature or directrix. One of the well known surfaces of this character, used very generally, as an example in discussing the properties of the surface, is the helical convolute, so named because it is generated by a line moving so that it is tangent to a helix.*

128. The helix has already been defined as the curve traced by a point which moves at a uniform rate around a cylinder, and at the same time at a uniform rate in a direction parallel to the axis of the cylinder, and, since these relations are directly proportional relations, a curve, plotted between them in coordinates, will be a straight, line, so that the helix is also the shortest line which can be drawn on a cylinder, between two points that lie neither upon the same right section, or upon the same right line element.

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*This surface is also called a developable helicoid, because it is the only helicoidal surface which can be developed.

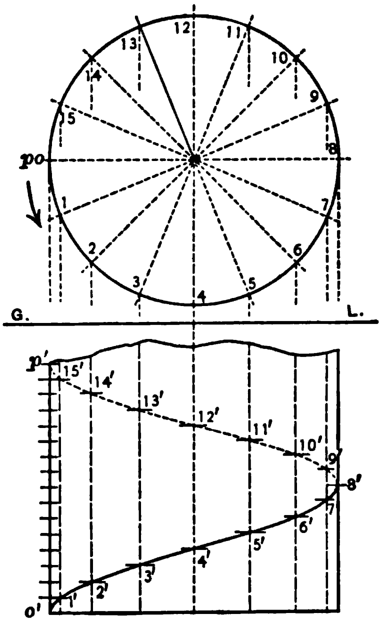

FIGURE 63.

To study the curve in detail, see Figure 63, which shows it traced on a circular cylinder, in third angle projection. Assume a point to be at O, and to move around the cylinder in the direction of the arrow through equal distances, 1, 2, 3, 4, etc. Let it also move up the cylinder through any given distance, until, after it has completed one revolution, it reaches a position P directly above O. The distance OP is known as the pitch of the curve. Now, as both motions are uniform, the point will travel to 1, which is one-sixteenth of the circumferential distance, in the same time that it travels one-sixteenth of the distance of OP toward P, and to 2, which is one-eighth of the circumferential distance, as it goes one-eighth of the distance OP toward P, and so on. Hence to plot the curve, divide the circumference of the base into any convenient number of equal parts, and the pitch into the same number of equal parts. By noting the points of intersection of the perpendiculars to the G.L., through the divisions of the circumference of the base, and parallels to the G.L., through corresponding divisions of the pitch, points of the curve may be found.

Certain peculiarities of this curve deserve to be noted. (1) It is tangent to the contour elements of the cylinder at points o' and $8'$ and p'. (2) It changes curvature at points $4'$ and $12'$, midway of the contour elements. (3) A tangent to the curve at $4'$ and $12'$ shows, by its angle with the projection of the base of the cylinder, the angular pitch of the curve. i. e., the ratio of the linear pitch to the circumferential distance. (4) The curve is sharpest at o', $8'$ and p' and gradually grows straighter until at $4'$ and $12'$ it is straight for a very short distance. (5) It is symmetrical, in parts, with respect to the projection of the axis of the cylinder, and to lines perpendicular to the projection of the axis, so that the curve from o' to $4'$ is a unit, which is repeated throughout the path of motion of the point.

The surface of the helical convolute is represented by drawing its helical directrix, at least two limiting elements, and its intersection with a plane, as a coordinate plane.

129. To represent a helical convolute with its axis perpendicular to H.

***Analysis:**—*The helical directrix will appear in both projections as described in Section 128. The V projection will be limited by tangents to the helical directrix at the points of changing curvature. The H projection of the surface will be as follows: The tangents to the helical directrix will be projected as tangents to the circle of the H projection. Since the pitch is constant, any tangent will make a constant angle with the H plane and is the hypotenuse of a right angled triangle, of which the H projection of the tangent, between the curve and the H plane, is one side, and the projecting line on the H plane, of any point of the tangent, is the other side. Moreover, from the fact that the tangent has the pitch of the directrix, at the point of tangency, and from the directly porportional relation in the motion of the point, in generating the curve, the H projection of a tangent is the length of the developed, or unrolled portion of the directrix or circle in H projection. Hence, if a series of tangents are drawn to the directrix, in H projection, and are made respectively equal in length to the circumferential distance between the H trace of the directrix and the respective points of tangency, the extremities of the tangents will lie upon an involute of the circle. The intersection of the helical convolute, by the H plane, or plane perpendicular to the axis, is therefore the involute of a circle.*

Construction:—Let the cylinder and the helical directrix be given as in Figure 64. At points 1, 2, 3, etc., of the V projection conceive tangents to be drawn to the

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*Which is an analysis for special conditions.

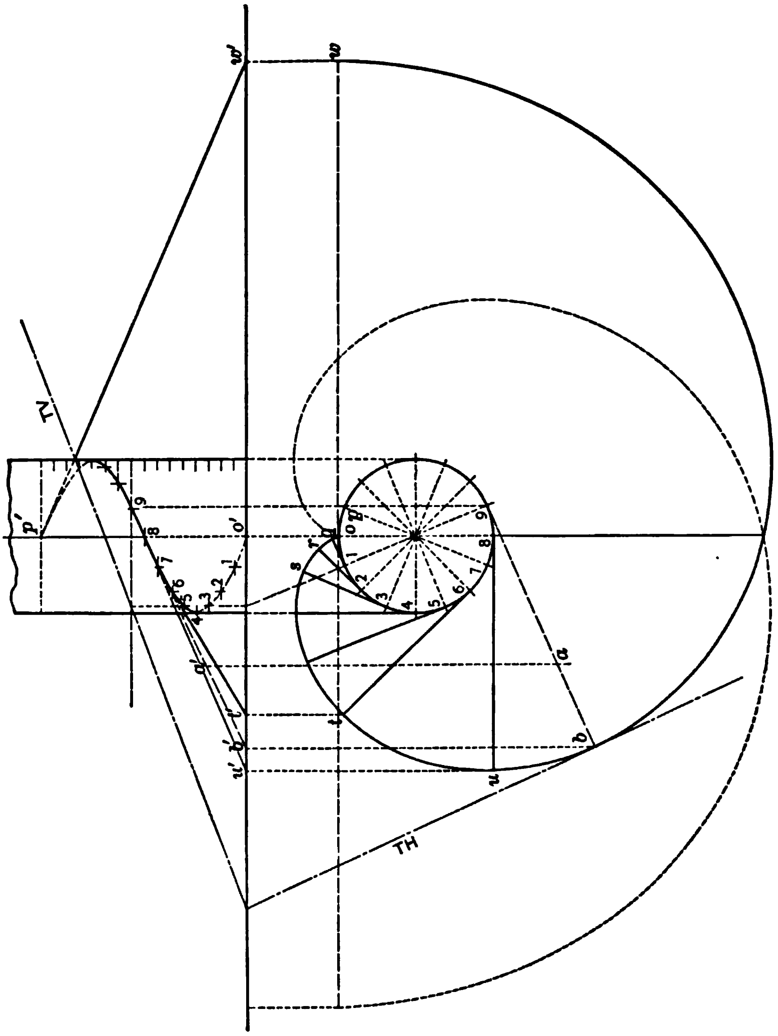

FIGURE 64.

curve. Since the traces of these tangents with the H plane cannot be obtained with accuracy, in this projection, derive them by first getting the corresponding projection of the traces as follows: O is the intersection of the directrix with H. Proceeding from O counter clockwise, draw the H projections of a series of tangents to the base in H, and make the length of each tangent equal to the rectified arc between the point of tangency and the point O giving the points Q, R, S, T, U., etc. Join these points with a smooth curve which is the curve of intersection of the surface with the H plane, and the latter would be called its base.

Now since the tangents are not limited in length by their traces and points of tangency, the surface will be continuous below the H plane; a portion of that part of it which is below is shown in dotted line.

If a plane parallel to H were to intersect the surface at a distance above H that 3 is above the G.L., then the curve of intersection would be an equal curve to the one in H but it would have a cusp at 3 instead of at O.

The elements are the lines of greatest declivity of the surface with respect to any plane perpendicular to the axis, and the surface itself is one of equal declivity with such a plane, in this case with the H plane.

131. To assume a point upon the surface of a helical convolute, and to pass a plane tangent to the surface to contain it.

Analysis:—*Since a point lies upon an element of the surface, assume either projection of the point, and draw an element through it by drawing a tangent to the helical* *directrix. The corresponding projection of the point will lie upon the corresponding projection of the element.*

In accordance with the definition of a tangent plane to a single curved surface, the required plane will be that of an element of contact through the given point, and a tangent to the curve of intersection of the surface with any plane. Hence, draw an element and through the trace of that element with any secant plane, draw a tangent to the curve of intersection of the surface with that secant plane. The plane of these two lines is the required plane. If the secant plane is a coordinate plane, then the tangent to the curve of intersection with the secant plane is at once a trace of the required plane.

Construction:—Let the surface be given as in Figure 64. Assume a point *a* on the surface. Draw an element through it to pierce H in the curve *s t u*. From *b'*, the V projection of this trace, draw a tangent to the V projection of the helical directrix and thus obtain *a'*.

Through *b* draw a tangent to the curve *s t u* and this is the H trace of the required tangent plane. With the H trace, and the elements BA or the point A in the plane, find the V trace.

132. To find the curve of intersection with a helical convolute of any secant plane oblique to the axis.

Analysis:—*According to the rule previously given for finding the curve of intersection of a plane with any single curved surface, a series of auxiliary secant planes may be chosen to cut lines from both. The most convenient planes, however, will probably be the projecting planes of the elements of the convolute Where these elements pierce the* *given plane, will give points upon the curve of intersection. Hence, by Section 57, find the piercing points of any series of elements with the secant plane, and join them by a smooth curve.*

If auxiliary planes are passed parallel to the plane of the base of the cylinder of the directrix of the convolute, they will cut the surface in involutes of circles equal to those cut from the surface by the plane of the base. The intersections of these involutes with the respective lines cut from the given plane, will be points on the curve of intersection required. And this method of solution may be employed, when it is found convenient to make a template to easily transfer the different involutes. The origin of the involute, or point of contact with the cylinder of the convolute, for any one auxiliary plane, will be where the trace of the auxiliary plane cuts the helical curve upon the cylinder.

133. To pass a plane tangent to a helical convolute and to contain a point outside.

Analysis:—*The tangent plane will contain a line through the given point, which will be tangent to a curve of section of the surface by a plane containing the point, and it will also contain an element of the surface passing throngh the point of tangency of this line. Hence, pass a secant plane through the point to cut the surface in a curve; draw a tangent to this curve, through the point. Also draw an element of the surface passing through the point of tangency, the plane of these two lines is the plane required.*

Construction:—Let the helical convolute be given as in Figure 65, with the point P as shown.

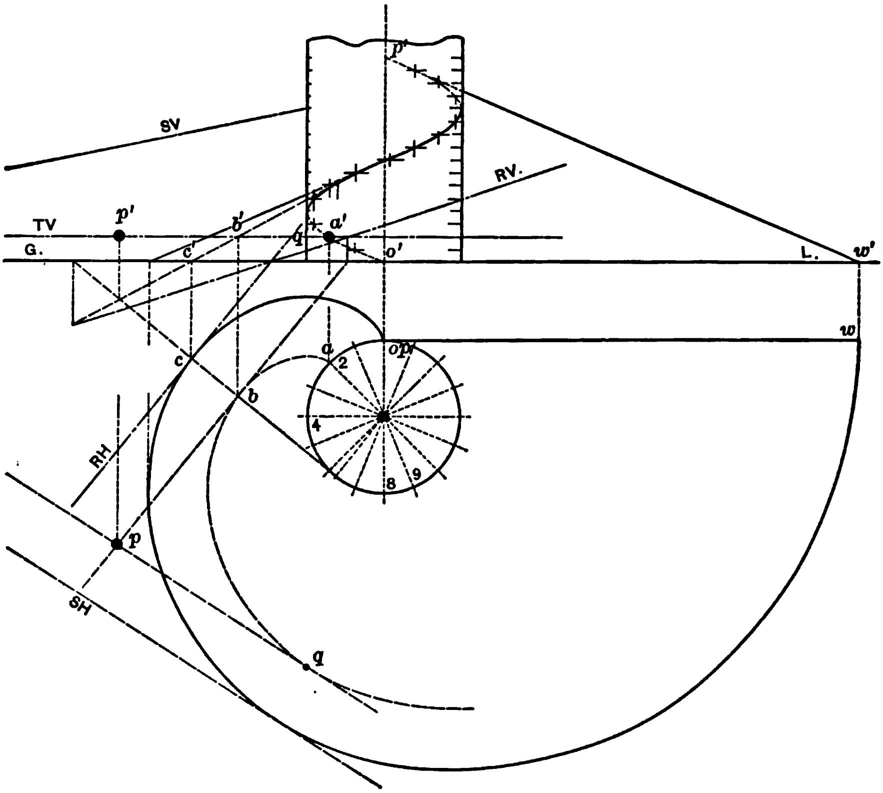

FIGURE 65.

For convenience pass a plane T through P parallel to H. Its V trace is parallel to the G.L. and the point A in which it cuts the helical directrix is the origin of the involute of a circle in which the surface cuts this plane. Through the point *p* draw a tangent to this involute as *p b;* its V projection is parallel to the G.L. and coincides with TV; through the point of tangency B, draw an element of the surface, BC; C is its H trace. The H trace of the required plane R goes through C and is parallel to the tangent *pb*. The V trace goes through the V trace of the element BC, and also that of the tangent PB. As two tangents can be drawn from the point P to the involute so there can be two tangent planes to the convolute surface through the point P. S will be the other plane as shown, containing the tangent PQ.

134. To draw a plane tangent to a helical convolute and parallel to a given line outside.

Analysis:—*The required plane contains a line parallel to the given line. Now, a plane passed through the given line and conceived to be parallel to the required plane may also be thought of as tangent to a right cone with base perpendicular to the axis of the convolute, and whose elements make the same angle with the plane of the base, that the elements of the convolute do with a plane perpendicular to its axis. Therefore, choose any point of the given line to be the apex of a right cone whose base shall lie in a plane which is a right section of the helical convolute. Pass a plane through this line and tangent to the cone, (by Section 118). The required tangent plane will have its traces parallel to this one and may be located by aid of the element of tangency as in the preceding problem.*

Construction:—Let the convolute be given as in Fig. 66 and the line as AB. It is seen from the element of the convolute, OP, which is parallel to V, that the angle of the elements with the H plane is a, hence, through any point of AB, as B, draw a right circular cone whose base is in H and whose elements make an angle of a with H. Then, through AB pass a plane T tangent to the cone. Its H trace goes through C, the H trace of AB, and its V trace

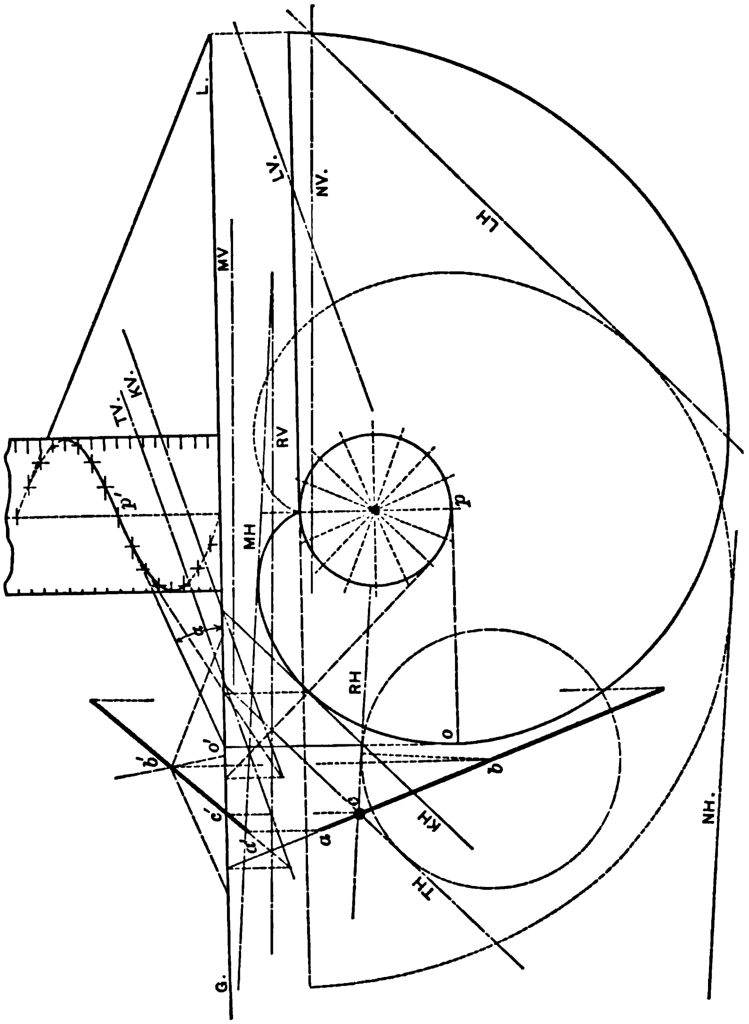

FIGURE 66.

contains the V trace of the line AB. Another plane R can be drawn tangent to the cone to contain the other tangent to the base from C.

Since the base of the auxiliary cone has been taken in H, the H traces of the possible parallel tangent planes to the convolute will be tangents to the curves of intersection of the surface with H as shown. There can be two planes K and L parallel to T and two planes M and N parallel to R. The V traces are obtained by one of the usual methods.

135. To develop the surface of a helical convolute.

Analysis:—*Assume, for simplicity in demonstration, that the convolute is one whose axis is parallel to V and perpendicular to H as discussed in Section 130, and let the curve of intersection of the surface with H, be called the base.*

Since the helical directrix makes a constant angle with the plane of the base, (Section 130), and since the elements retain the same length in the development, and also the angles between consecutive elements are equal, the length of the base remains constant, and the helical directrix develops into a curve of uniform curvature. This curve will be a circle, and the developed base will be the involute of this circle.

The mathematical relations for determining the radius of curvature of the developed helical directrix may be stated as follows:

$$R = \frac{a}{\cos^2 \alpha}$$

when a = radius of the cylinder on which the helix is wound and α the angle a tangent to the helix, makes with a plane of right section.

But the radius may be obtained graphically as follows: At the point p' of Fig. 66, draw a horizontal line, at the intersection of the tangent $o'p'$ with the contour of the cylinder draw a normal to the tangent to intersect the horizontal line through p'. The distance from the intersection to p' is the radius of the circle representing the developed helical directrix. The developed surface will be limited by this circle and its involute.

CHAPTER VI.

WARPED SURFACES.

136. Warped Surfaces.

Warped surfaces have already been defined as those generated by a line moving so that no two of its successive positions intersect each other, which is equivalent to saying that they do not lie in the same plane.

Any two elements of one system of generation are what is known as *windschief*. The motion of a line, in generating these surfaces, while infinite in variety is, in general, however, specified as being controlled by lines, curved or straight, or by surfaces or both. That is, it moves so as to touch a line in a certain way, or a surface, or be parallel to the respective elements of a surface. One of the laws in illustration is that of a line which moves so as to touch three right line directrices which themselves do not intersect. Another is one in which a line moves so as to touch a given right line and a given curve and at the same time be parallel to a given plane, or still another to make a fixed angle with the given right line.

To summarize the limiting conditions, a line may move (1) so as to have its successive positions parallel to a plane; (2) so that its successive positions are parallel to the elements of a cone*; (3) so that its successive positions intersect a given line.

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*The cone is not limited to a right circular cone.

Under (1) we have (*a*) two right line directrices, (*b*) two curved directrices, (*c*) one straight line directrix and one curved line directrix. Under (2) we have (*a*) two straight line directrices, (*b*) two curved directrices, (*c*) one straight line directrix and one curved line directrix. Under (3) we have a unique class in which there are two curved directrices and one straight line directrix.

The classification may be tabulated as follows, with examples of the different kinds of surfaces:

137. Table of classification of warped surfaces.

	Directrix	Directrices	Examples
Warped Surfaces	Plane	2 straight lines	Hyperbolic paraboloid
		2 curved lines	Right helicoid
		1 straight line 1 curved line	Conoid
	Cone	2 straight lines	Hyperboloid of revolution*
		2 curved lines	Oblique helicoid
		1 straight line 1 curved line	Elliptical hyperboloid
	Line	2 curved lines	Oblique helicoid of varying pitch

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*The hyperboloid of revolution will, for convenience, be discussed under surfaces of revolution.

138. The Hyperbolic Paraboloid.

If a lines moves so that it touches two windschief right lines and remains at the same time parallel to a given plane, the surface generated is know as the hyperbolic paraboloid, so designated because sections of the sursurface are either right lines, hyperbolas or parabolas.

If two lines, not in the same plane, are intersected by a series of parallel planes, the lines are divided into proportional parts by the planes, from geometry, and if the corresponding points of division on each are connected they constitute the elements of a hyperbolic paraboloid.

Conversely, if any two lines not in the same plane are divided into proportional parts, the right lines joining the corresponding points of division on each will lie in parallel planes and be the elements of a hyperbolic paraboloid, the plane directer of which is parallel to any two of them.

Thus let AB and CD, Fig. 67, be any two non-intersecting lines. With any radius as *a-1* divide *ab* into any number of equal parts; with any other radius as *d-6* divide *d-c* into the same number of equal parts, find the vertical projections of these points of division. Join the corresponding points of division on these lines as *1-d*, *2-12*, *3-11*, etc. They are the elements of the surface, and it is seen that the curve which is the envelope of its tangents is the parabola. For from geometry, any element as *9-5*, being a tangent to the parabola, divides the two other tangents AB and CD proportionally, a well known property of the parabola. The lines AB and CD are the two linear directrices of the surface and are shown here as oblique to the coordinate planes.

The characteristic properties of this surface are best

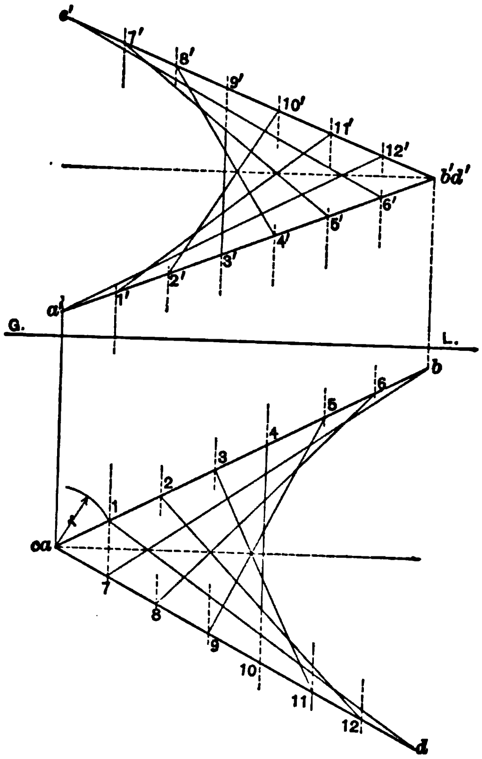

FIGURE 67.

brought out by projecting the elements of the surface upon two coordinate planes which are not necessarly at right angles to each other, one parallel to the two rectilinear directrices which we will call Q, the other parallel to the elements of the one system of generation, or in other words the plane directer of the system which we will call P. Let Fig. 68 represent these planes seen edgewise, in other words projected upon an end vertical plane. The projecting lines of any point M will be parallel respectively to the

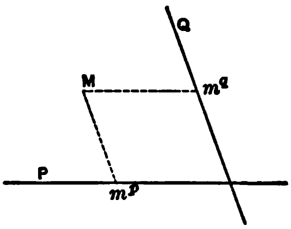

FIGURE 68.

coordinate planes — although not perpendicular to each other,—and parallel also to the end plane, so that when the planes P and Q are unfolded, the projecting lines will constitute one and the same perpendicular to the G. L.

139. *THEOREM XII.

The projections of all the elements of one system of generation of a hyperbolic paraboloid, upon the plane directer intersect each other in a point, known as a 'point of concourse.'

Proof—Let Figure 69 represent the projections of the two directrices AB and CD, upon two inclined coordinate planes, Q and P, respectively, and let P be taken as a horizontal plane. Since the plane Q is parallel to both of the directrices, their P projections will be parallel to the G.L., and they may, with this proviso, be assumed anywhere as shown. Let the point of intersection of the Q projections of AB and CD, be denoted by y^q— merely an arbitrary point and not standing for the projection of any point in space.

Assume any plane parallel to P, its vertical trace being TQ. It will pierce the directrices in two points, respec-

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[*] After Watson's 'Descriptive Geometry', page 41.

tively, E and F, which, if connected by a line, will constitute an element of the surface. Drop a perpendicular to the G.L. through y^q, and let its intersection with f^pe^p be called x^p also its intersection with the G.L., be called *1*, and with the P projection of the directrix AB as *2*, and with TQ as *3*. And let the intersection of a perpendicular to the G.L. through f^q with the P projection of one of the directrices be called *4*. From the similar triangles e^px^p2. e^pf^p4, b^qy^q1, e^qy^q3, y^q1d^q, and y^q3f^q, we have—

$$\frac{x^p2}{f^p4} = \frac{e^p2}{e^p4} = \frac{e^q\,3}{e^qf^q} = \frac{b^q1}{b^qd^q}$$

but the ratio $\frac{b^p1}{b^qd^q}$ is independent of the position of the generatrix FE, that is, the cutting plane T, hence

$$\frac{x^p2}{f^p\,4} = \frac{b^q1}{b^qd^q} = \text{constant}$$

and x^p2 is a constant for all positions of the generatrix.

By similar reasoning, it also may be proved that the projections of the other system of generation, upon the plane directer Q, pass through a point on that plane, which point is the intersection of the Q projections of the directrices AB and CD, denoted y^q.

140. THEOREM XIII.

The section of a hyperbolic paraboloid by a plane parallel to the two rectilinear directrices is a straight line.

Proof—Let the surface be represented as in Fig. 69, and assume R to be a plane parallel to Q, the plane which in turn is parallel to the two directrices. The

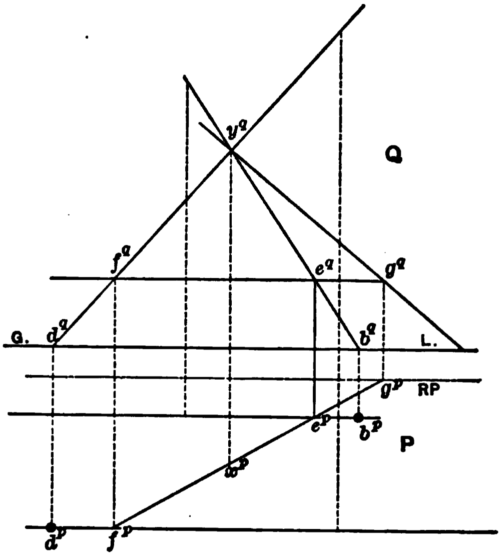

FIGURE 69.

generatrix FE meets this plane R in the point G. From similar triangles we have:

$$\frac{f^p e^q}{e^p g^p} = \frac{f^q e^p}{e^q g^q}$$

But we see that this ratio would be true for any position of the elements EF, that is the ratio $\frac{f^q e^q}{e^q g^q}$ is a constant and the locus of the various intersections of elements of the surface with any plane R would be a straight line which goes through the point y^q.

Corollary 1:—*For every hyperbolic paraboloid there are two systems of generation.* For since any plane parallel to Q will cut the surface in a line whose P projection is parallel to the G.L., the plane Q will be the plane directer of the system of such intersections or elements and the projections of these elements upon the plane Q pass through a common point y^q, just as those belonging to the other system pass through x^p with P as the plane directer.

Corollary 2:—*The generatrix of one system in a hyperbolic paraboloid intersects every element of the other system.* For the projection of any two generatrices, one of each system, upon either coordinate plane P or Q will intersect each other in a point which can be the projection of but one point upon the surface. Hence, any two elements of one system may be taken as the directrices for the other system. It follows also that through any point of the surface two lines may be drawn which are, respectively, the elements of each system of generation.

Corollary 3:—*No two generatrices of the same system in a hyperbolic paraboloid intersect each other* for we have seen that the projection of one system upon one of the coordinate planes P and Q are lines parallel to the G. L. and in the corresponding projection they are oblique and make different angles with the G. L.

Corollary 4:—*The hyperbolic paraboloid may be defined as the surface generated by a line which moves so as to touch three other non-intersecting lines* all of which are parallel to a common plane and divide said lines into proportional parts for since an element of one system intersects every element of the other system the surface may have three rectilinear directrices instead of two rectilinear directrices and a plane directer.

It was noted in Theorem XIII that for the first system of generation discussed there was a point in the projection of the elements upon the plane directer which is common to the projection of all the elements on that plane, y^q, and for the second system, similarly, a point in the projection of these elements upon their plane directer, which is common to all these elements, x^p. These points are known as '*points of concourse*'. It may, moreover, be seen that each point of concourse, as a point in one system, is the projection upon that plane directer of an element of the surface belonging to the other system of generation which coincides with the projecting line of that element upon the plane. For example, referring to Figure 68, the line Mm^p is the projection of such an element parallel to Q, upon the plane P, and Mm^q is the projection of an element parallel to P upon the plane Q.

The trace of the hyperbolic paraboloid upon the plane directers P and Q is found as follows: —The P trace goes through the P traces of the directrices AB and CD, see Fig. 70, and since an element of the surface is projected upon Q in the point y^b that being also its Q trace the Q trace of the plane of the hyperbolic paraboloid goes through that point SQ and SP will be its respective traces. If the traces did not meet in the G. L. within the limits of the drawing, then the Q trace of any auxiliary element of the surface could be found and the required traces located.

141. To assume a point upon the surface of a hyperbolic paraboloid.

Analysis:—*Assuming either projection of the point, draw through it any element of one system of generation. The corresponding projection of the point will be found upon*

the corresponding projection of the element, and the corresponding projection of the element can be found by its intercepts upon the directrices.

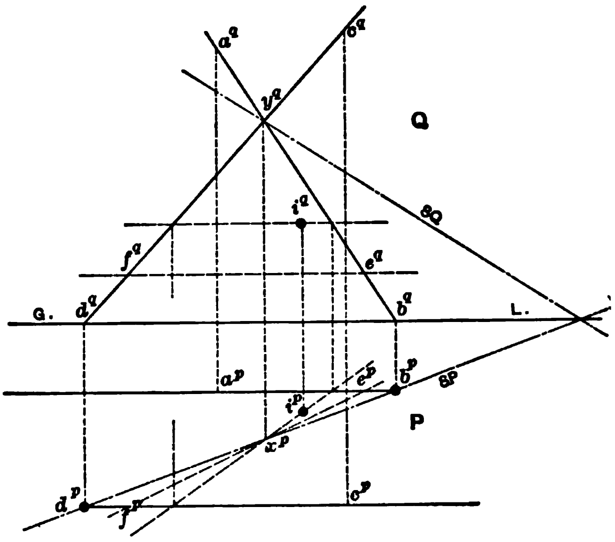

FIGURE 70.

Construction: — Let the surface be represented, as in Fig. 70 and let i^p be the projection of an assumed point. draw through i^p and the point of concourse, x^p, an element of the system of generation which is parallel to P. Find by its intersection with the P projection of the directrices, the Q projection of this element. i^q is found at the intersection of a perpendicular to the G. L through i^p and the Q projection of the element.

If i^q had been assumed, draw the Q projection of an element through the point belonging to the system which is parallel to P by making its Q projection parallel to the G.L. The intercepts with the directrices will give the P projection and from that the P projection of the point may be found upon the P projection of the element through the point.

Note:—An element drawn through $i^q y^q$ will, in P projection, pass through i^p, and be parallel to the G.L., and this element may equally well be used to obtain the corresponding projection of the assumed point.

Tangent planes to warped surfaces have the same peculiarities as those to other surfaces which have already been discussed, that is, they are the locii of lines which can be drawn tangent to curves cut from the surface. An element of a surface which has straight line elements is a convenient line of the plane to obtain. In warped surfaces a tangent plane while it may contain an element or elements of the surface will only be tangent in general, at a point of the surface, for the successive elements are non-intersecting, and a plane passed through one will intersect the next consecutive one in a point. Therefore the tangent plane at a given point is derived conveniently by an element through the point and a tangent to the curved section through the point. If the surface is doubly ruled like the hyperbolic paraboloid, the tangent plane will contain the two elements of the surface, one of each system of generation, intersecting in the point.

142. To pass a plane tangent to a hyperbolic paraboloid at a point on the surface.

Analysis:—*The tangent plane must contain an element of the surface; but through every point of the surface, there pass two elements one belonging to each system of generation, hence the tangent plane will contain both and be* *tangent to the surface, only at a point. Therefore, through the given point, draw an element of each system of generation. The plane of the elements is the plane required. The curves cut from the surface by the tangent plane, are the two elements lying in the tangent plane.*

Let any generatrix of the system P, that is the one of which P is the plane directer, revolve about the point of concourse, x^p, occupying different possible positions in the surface, see Fig. 71. It will assume positions $x^p i^p$, $f^p e^p$, $d^p b^p$, etc., respectively. When it occupies a position parallel to the G.L., its Q projection would be at an infinite distance below the G.L. If it continues to revolve, it would reappear again from an infinite distance above the G.L. on the plane Q. When its P projection is perpendicular to the G.L., its Q projection is the point y^q.

Therefore if through any point in the plane directer of one of the systems we draw lines parallel, respectively, to the elements of that system, they will occupy all possible positions around that point, and the one infinitely distant will be parallel to the G.L. And since there is in each system an element infinitely removed and parallel to the G.L., it is evident that an element can be found to make any assigned angle with the line of intersection of the plane directers.

The two elements, one of each system, whose projections are the points of concourse, respectively, intersect each other in a point of the surface known as the vertex.

The two diametral planes of the surface are known as those respectively which (1), are perpendicular to each other and go through the vertex perpendicular to the G.L., or intersection of the plane directers, and (2), go through the vertex and bisect the dihedral angle between the two plane directers.

If the surface is projected upon the coordinate planes in such a manner that H is parallel to the first mentioned diametral plane and V is parallel to the second mentioned diametral plane, we will obtain projections which will facilitate the drawing of elements of the surface, tangent planes, sections, etc.

143. (a) To draw the projections of the hyperbolic paraboloid upon planes parallel, respectively, to the diametral planes, (b) to find the sections of the surface made by planes parallel to the coordinate planes, (c) to construct a tangent plane at any given point.

Analysis:—*The analysis of the first part of the problem is analogous to the explanation given in Section 139 and need not be repeated.*

Any secant plane will cut either straight lines, hyperbolas, or parabolas from the surface. Secant planes parallel to H, namely, perpendicular to the plane directers, will cut hyperbolas of equal nappes, and one will cut two straight lines. Any secant plane parallel to V, namely, parallel to the plane bisecting the dihedral angle between the plane directers will cut the surface in a parabola.

Any tangent plane through a given point is obtained by getting the traces of the plane of the two elements one of each system intersecting in the point

Construction:—See Figure 71. The H plane is taken perpendicular to the two plane directers P and Q, and the V plane is taken parallel to the plane bisector of the

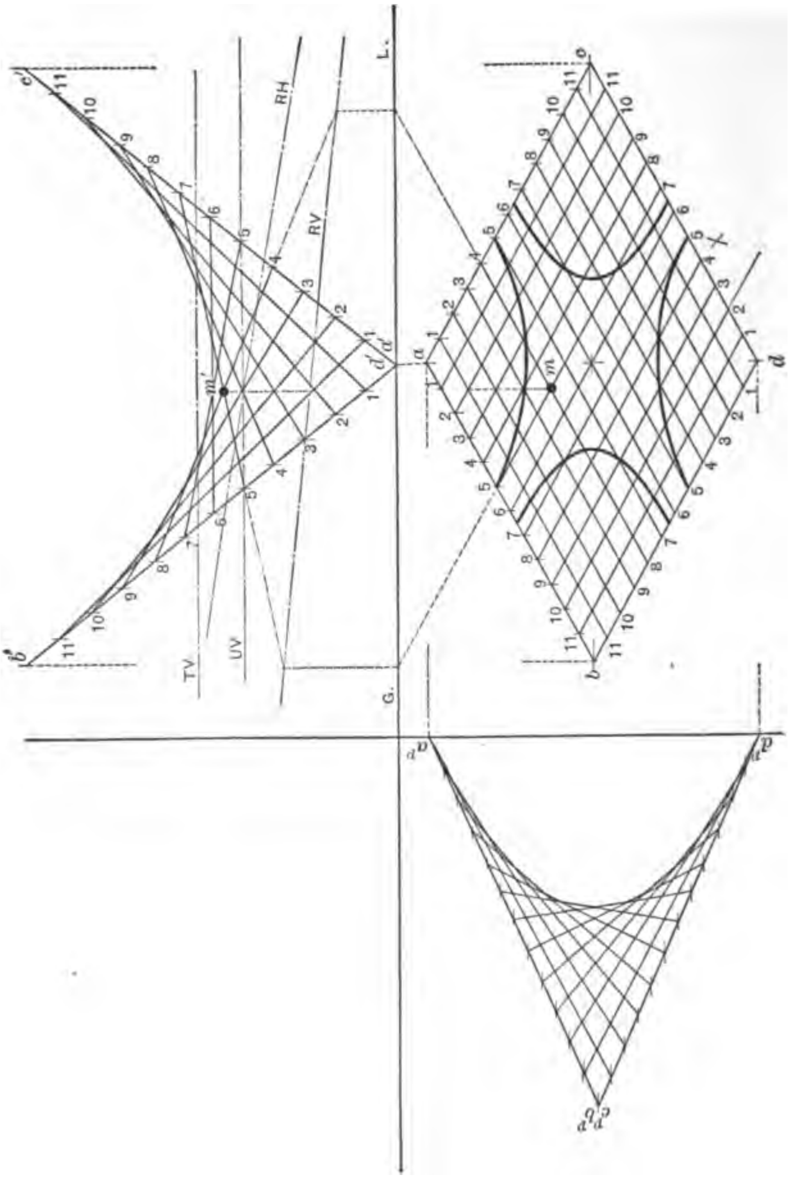

FIGURE 71.

dihedral angle between P and Q. It can be seen that the projections of both systems of elements upon the H plane will be parallel to each other, respectively. AB and CD are the two directrices. These are divided proportionally into twelve equal parts and the corresponding points of division joined, giving as elements of the one system 1-11, 2-10, etc. By joining AC and BD, and simliarly dividing these into twelve equal parts we get elements 1-11, 2-10, etc., of the other system of generation. These are shown also projected upon an end plane, for convenience revolved into H. The directrices AB and CD in V projection are for convenience assumed as equally inclined to the G.L.

A parabola is, in the V projection as in Figure 71, the envelope of the tangents, and is the curve of intersection with the surface of a plane parallel to V, its points corresponding to those in which the elements, in H projection, intersect the line *bc*. For, referring to the V projection and the tangent 7-5, together with AB and CD.

$$\frac{c'5}{5a'} = \frac{d'7}{7b'}$$

Hence, the tangent 7-5 divides the tangents AB and CD into proportional parts, a well known property of the parabola.

Assume a horizontal plane to cut the surface, shown by its trace TV. Noting the points in which the respective elements pierce this plane, and getting their respective H projections, it is found that they lie on a hyperbola of two nappes, symmetrical with respect to the line connecting *a* and *d*. Assume another V plane, shown by its trace UV, and similarly plotting the traces of elements with it, we find that the curve of intersection is another hyperbola, with two nappes symmetrical with the line connecting *b* and *c*. The asymptotes to the hyperbola can be found by taking a horizontal section of the surface, and passing through the vertex of the parabola in V projection.

A section of the surface, by a plane oblique to H, would give hyperbolas of unequal nappes.

Let it be required to draw a tangent plane, at the point M, the intersection of the elements 7-5 and 8-4, one belonging to one system of generation, and the other belonging to the other system. The plane of these two elements is the plane required, shown by its traces RV and RH. It can be seen, that it is also a secant plane of the surface, being tangent only at the point M.

The vertex of the surface is the point of intersection of two elements, one of each system, whose plane is perpendicular to both plane directers, i.e., perpendicular to their line of intersection. Such a point is the intersection of elements 6-6 of each system in Figure 71.

The axis of the surface is a line through the vertex, parallel to both plane directers, i.e., parallel to their line of intersection.

If the angle between the plane directers is a right angle, the surface is known as a right or *rectangular hyperbolic paraboloid.* If the angle is not a right angle, the surface is known as an *oblique hyperbolic paraboloid.*

144. To assume a point upon the surface of the hyperbolic paraboloid as in Section 141.

Analysis:—*The horizontal projection of the point may be assumed, and an element of the surface drawn through it from whence, the V projection may be obtained. The V* *projection of the point may be assumed; in this case pass a horizontal secant plane through it, to cut a hyperbola from the surface, whence the H projection of the point will be found upon the hyperbola. The assumed V projection stands for two points upon the surface, equally distant in front of and behind the vertex.*

145. To pass a plane through a given right line and tangent to a hyperbolic paraboloid.

Analysis:—*The tangent plane contains an element of the surface, in addition to the given line. Therefore, through the piercing point of the given line, and the surface, draw an element of the surface; the plane of these two lines is the plane required. It will be tangent to the surface at some point of the element.*

The pilot or cow-catcher of some locomotives is built upon the lines of a hyperbolic paraboloid. The prows of some boats illustrate it also.

146. The Conoid is a warped surface having a straight line, and a curved line directrix, and a plane directer.

A right conoid is a conoid in which the plane directer is perpendicular to the straight line directrix and the plane of the curvilinear directrix. It may be exemplified by a line which moves so as to touch a straight line and a given circle, the given line being parallel to the plane of the circle, and the plane directer, perpendicular to both, whence we have a surface as represented in Fig. 72. The plane directer is an end vertical plane.

A right helical conoid is a conoid in which the curvilinear directrix is a helix, and the straight line directrix the axis of the helix. It is exemplified in the face of a square threaded screw.

An oblique helical conoid is a warped surface in which the generatrix, touching the helix and its axis, is oblique to the latter. It is exemplified in the V threaded screw.

147. To draw a conoid having a circular directrix, and a straight line directrix parallel to the plane of the circle and a plane directer perpendicular to both; to assume a point on it, and to draw a tangent through the point.

Analysis:—*Assume one coordinate plane parallel to the plane directer, and one perpendicular to it, and also one parallel to the circular directrix. The projection of the elements upon the second plane will be parallel to the G.L., and their projections upon the first will go through a common point which is the projection, upon that plane, of the straight line directrix.*

To assume a point on the surface, assume either projection and draw an element through the point.

To pass a plane tangent to the surface at any point, draw an element through the point, and it will be contained within the tangent plane. The tangent plane will also contain a tangent, at the point of contact, of a curve of section of the surface made by any secant plane. The plane of these two lines is the plane required.

Construction:—The conoid is shown in projection in Fig. 72. The circular directrix is placed in H and the straight line directrix perpendicular to V, which is the plane directrix. For convenience, the elements are taken as touching the circular directrix at equally distant points, 1, 2, 3, 4, etc. $o'p'$ is the vertical projection of the direct-

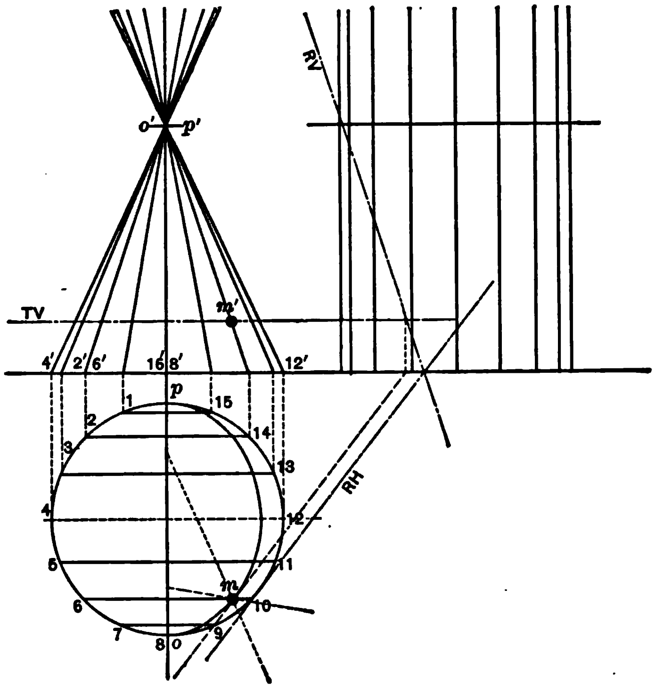

FIGURE 72.

rix, seen as a point. This directrix is also an axis of the surface. Another axis, more generally considered as the axis, is a line through the center and perpendicular to the plane of the circular directrix.

Planes perpendicular to this axis cut the surface in ellipses.

To assume a point on the surface:—Assume either projection, say the H projection as *m*. Draw an element through the point, and by its vertical projection, obtain the V projection of the point. The assumed projection of the point, in any case, would stand for two points on the surface; if the H projection, then a point on either nappe, if the V projection then a point upon either the front or back of the surface.

To draw a tangent plane to the surface, say at the point M. Draw an element of the surface through the point; it will be contained within the tangent plane, Pass a plane through the point, to cut a curve from the surface, preferably a plane parallel to H as T. The tangent to the ellipse of section at M will also be a line of the plane. The V trace of the required plane, R, goes through the V trace of this line, and is parallel to the element of the surface, through the point of tangency. The H trace is parallel to the tangent to the curve of section.

This plane will not be tangent to the surface all along an element, because sections of the surface, at various points to intersect an element, through any given point, will not have tangents at those points of intersection which are parallel.

The foregoing example is only one of an infinite number of conoids.

The curvilinear directrix may be either a curve of single or of double curvature. The straight line directrix may occupy any position, with respect to the curvilinear directrix. For example, if, in a conoid of the class just described, the straight line directrix had been oblique to the circle, we would have had an oblique circular conoid.

A right elliptical conoid is one in which the conditions are the same as in the case just discussed, except that the curvilinear directrix is an ellipse.

148. The helicoid is a warped surface generated by a line which moves so as to touch two helical directrices,* having a common axis, and has its successive positions parallel, either to a plane directer, or to a cone directer.

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* A helix, as here used, is a term designating a helix described upon a right cylinder.

A right helicoid is a helicoid, in which the elements are parallel to a plane directer.

An oblique helicoid is a helicoid in which the elements are parallel to those of a cone, i.e., has a cone directer.

The axis of the helical directrices is known as the axis of the surface.

A helicoid of uniform pitch is a helicoid in which both helical directrices have the same pitch.

A helicoid of varying pitch is a helicoid in which the helical directrices have different pitches, hence, the motions of various points of the genertrix axially, vary, and the angles of the generatrix, with the axis, vary.

149. Figure 73 represents an oblique helicoid. One spire of the helical directrix, 1, 4, 7, etc., is shown, with its axis parallel to V, Elements of the surface may be drawn as follows: Assume the angle of the elements, with the axis of the helical directrix, to be 30°. Draw an element of the surface, which is parallel to V, and will show its true angle with the directrix. Such an element will be $e'a'$. Since the angle of the elements with the straight line directrix is constant, every point of the element will describe a helix of constant and equal pitch; therefore, the points of intersection of the elements, with the straight line directrix, will be distant from one another the amount of axial advance which the points of intersection of the elements, with the helical directrix, are one from another. Therefore, to draw any other element: Choose a point upon the helical directrix, as $6'$ through which the element is to pass, note its axial distance from the point 1, that is,

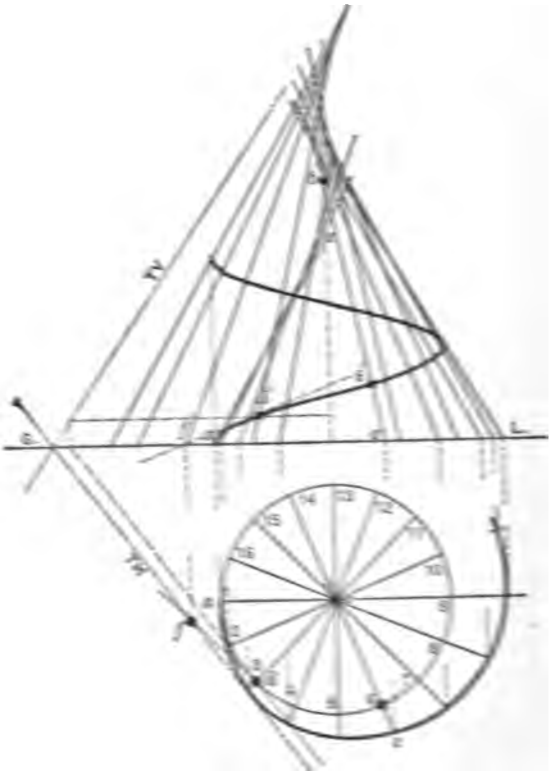

FIGURE 79.

in this case, the distance the point is from the H plane. Lay this distance off on the axis, measuring upward from the point *e* obtaining *b*, then *b e* is the V projection of the element; it intersects H in a point C, a point upon the curve of intersection of the surface with the H plane; other elements may be assumed in a similar manner.

The H projection of an element may be assumed as readily as the V projection, and from it the V projection, by the method just described.

150. To assume a point upon the oblique helicoid: Assume one projection of the point, as the H projection. *Draw the projection of the element through it to intersect the axis and the helical directrix; from the V projection of the latter point, note its axial advance from any other convenient point on the directrix, through which an element passes, and proceed as described in the preceding paragraph.

151. To pass a plane tangent to an oblique helical conoid, at a given point on the surface.

Analysis:—*The tangent plane will contain an element passing through the given point; it will also contain a tangent to a curve of section of the surface, through the point, which is a helix of equal pitch but lesser diameter than that of the directrix. But the tangent to the directrix, at the intersection of the element, is in general most convenient.*

Construction:—Let the surface be given as in Fig. 73, and let the point of tangency be D. Draw an element of the surface, through the point. The H trace of the tangent plane goes through the H trace of this element. A tangent DF, to the helical directrix, may next be drawn. The H trace of the tangent plane also goes through the H trace of this tangent. The V trace can then be found, D being a point in the plane.

152. To find the curve of intersection of an oblique helical conoid by any plane.

Analysis:—*The general conditions of surfaces being*

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*For convenience we will consider the helicoid as projected on H and V, when its axis is perpendicular to the former.

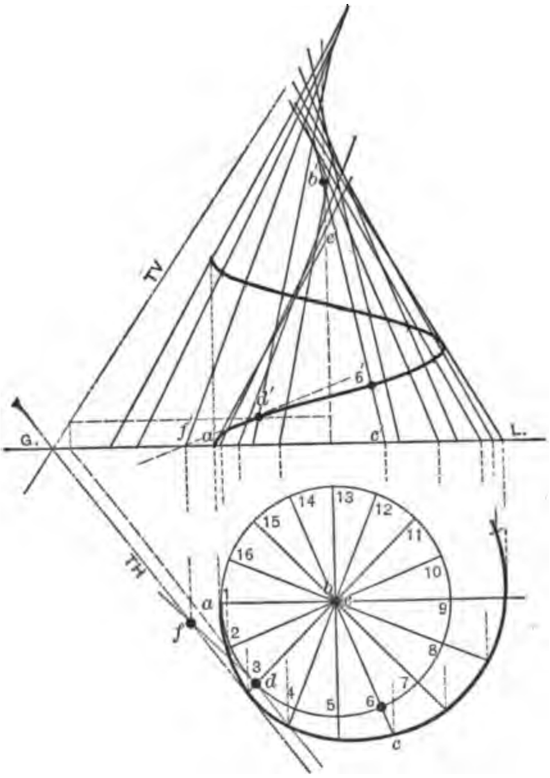

Figure 78.

in this case, the distance the point is from the H plane. Lay this distance off on the axis, measuring upward from the point e' obtaining b', then $b'6'$ is the V projection of the element; it intersects H in a point C, a point upon the curve of intersection of the surface with the H plane; other elements may be assumed in a similar manner.

The H projection of an element may be assumed as readily as the V projection, and from it the V projection, by the method just described.

150. To assume a point upon the oblique helicoid: Assume one projection of the point, as the H projection. *Draw the projection of the element through it to intersect the axis and the helical directrix; from the V projection of the latter point, note its axial advance from any other convenient point on the directrix, through which an element passes, and proceed as described in the preceding paragraph.

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152. To find the curve of intersection of an oblique helical conoid by any plane.

Analysis:—*The general conditions of surfaces being*

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*For convenience we will consider the helicoid as projected on H and V, when its axis is perpendicular to the former.

intersected by planes, can be observed here. The auxiliary secant planes, by being taken so as to contain the axis of the surface, will cut the latter in straight lines, the elements of the surface, and their intersections with the given plane, will give points upon tne curve of intersection.

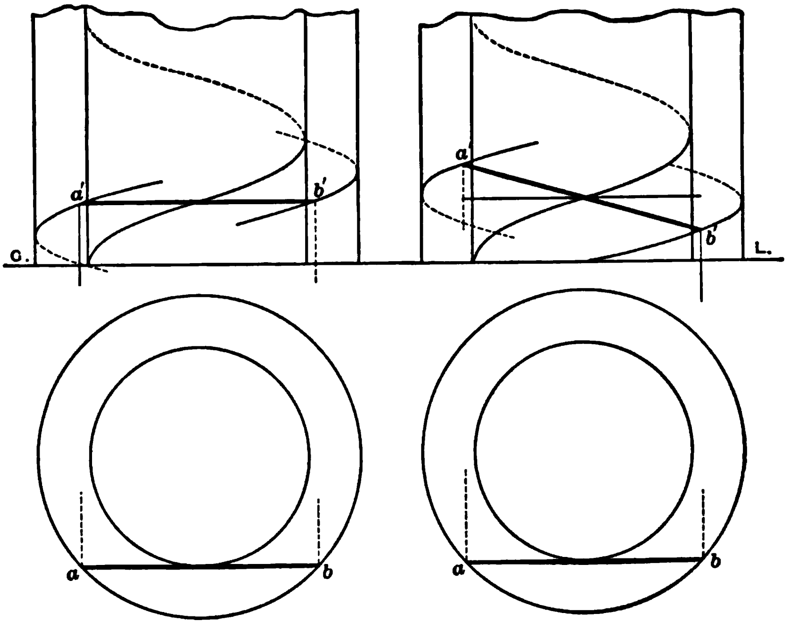

FIGURE 74 AND 75.

153. The right and oblique helicoids are shown in Figs. 74 and 75.

Figure 74 is the right helicoid. Figure 75 illustrates the oblique helicoid. The moving element is shown in each figure as the line AB. It is tangent, in each, to the inner cylinder, upon which the helix is described, although it cuts the helical directrix.

This surface, like many others, is not adequately represented by drawing a few positions of elements. It is necessary to take a limited element, and plot the paths traced by several points, on the element and draw the contour of which these paths are the envelope. If this is done, it is seen that the oblique helicoid has the form of an augur.

154. THEOREM XIV.

The trace of an oblique helicoid with any plane perpendicular to the axis of the helical directrix is an archimedian spiral.

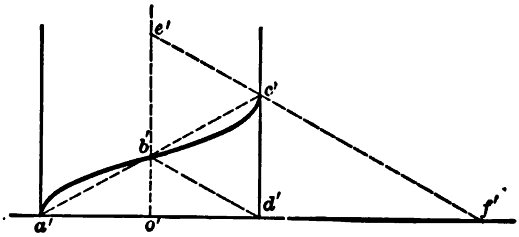

FIGURE 76.

Proof.—Referring to Figure 76, suppose an element *a′b′c′* to move half way around the cylinder. It will occupy a position *e′c′f′*, parallel to *b′d′*, since the pitch is constant. Since $b'c' = 2\ o'b'$, we will have $d'f' = 2\ o'd' = a'd'$. And, since the revolutions of points of the element are uniform, and the axial advance is also uniform, the trace with a plane perpendicular to *e′o′*, is an equable spiral, of which *d′f′* is the radial expansion, in half a revolution; and for any other position of the element, it is directly proportional to the axial advance, and, since these properties are identical with the archimedian spiral, the curve traced is identical.

155. To ascertain the point of tangency, of any given plane, with a oblique helicoid, along a given element.

Analysis—*Since the tangent plane contains but one element of the surface, it will pierce the other elements in points, which lie upon a curve, cutting the given element in the point of tangency.*

156. The hyperboloid of one nappe is a unique warped surface in that it is also a surface of revolution, and the only one of the latter class which is, at the same time, a warped surface. Since surfaces of revolution have special characteristics, controlling their treatment in projection, and as the hyperboloid has no particular characteristics, as a warped surface, beyond those which have already been discussed, it will be treated fully latter in connection with surfaces of revolution.

CHAPTER VII.

DOUBLE CURVED SURFACES AND SURFACES OF REVOLUTION.

157. Double curved surfaces.

Double curved surfaces are those which can only be generated by the motion of curves; they have no straight line elements, for, if they did, they would belong either to the single curved surfaces or warped surfaces. But the converse is not true, for, every surface generated by the motion of a curve is not a double curved surface. A double curved surface may be fully defined as one generated by the motion of any curve, such that no two consecutive points of the curve travel in straight line paths.

There is an infinite variety of double curved surfaces, since there is both an infinite variety of curves, plane and double curved, and an infinite variety of ways in which the curves may move to generate surfaces. But the most familiar double curved surfaces, and those of interest to the student, are also surfaces of revolution. No particular interest attaches to, or value to be gained by, the discussion of double curved surfaces, in general, except to note the following points:

Planes can be tangent to such surfaces only in a point, or isolated points.

A plane may be tangent to a double curved surface, and also intersect the surface in a curve, which may or may not contain the point, or points, of tangency.

A tangent plane, if it also cuts the surface, or a secant plane will intersect it in a curve, or curves, never in a straight line.

The law of generation of a double curved surface is not limited to the motion of any particular curve in any particular surface, for the curve formed by any secant plane, can be conceived of as one which could generate the surface, by moving and changing its shape, according to a law. The best definition would be that involving, at once, the simplest curve of section, and the simplest law of motion, to be decided by an inspection of the particular surface in question.

158. A surface of revolution is one generated by the revolution of a line about a right line as an axis. The revolving line may be a right line or it may be a curve, it may cut the axis or it may not.*

If the revolving line is a right line, it generates either the cone, when it intersects the axis or the only other surface of its class, known as a hyperboloid of revolution.

If the revolving line is a curve, it generates a double curved surface, and has no straight line elements. There may be an infinite variety of surfaces of revolution depending upon the form of the revolving curve, the simplest being the following: The sphere, formed by the revolution of a circle about one of its diameters; a cone, formed by the revolution of a right line about another right line which it intersects; a cylinder, formed by the revolution of a right line about another right line to which it is parallel; an ellipsoid, formed by the revolution of an ellipse about one of its axes;* a paraboloid, formed by the revolution of a parabola about its axis; a hyperboloid of revolution of two nappes, formed by the revolution of a hyperbola about its transverse axis, etc., the surface, in general, deriving its name from that of a curve which revolves.

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*A surface of revolution is not strictly limited to one formed by the revolution of a plane curve, although in general such are the ones considered.

It is obvious that any plane, perpendicular to the axis of a surface of revolution, will cut circles from the surface whose centers are in the axis, therefore, it is possible to conceive of such a surface being generated by the motion of a circle, such that the center moves along the axis of the plane of the circle while the latter changes its shape according to a law.

If planes are passed through the axis, they will cut equal curves from the surface, known as *meridian curves. That meridian curve which is parallel to a coordinate plane*, the axis being also parallel to the coordinate plane, is known as the *principal meridian curve.*†

In any surface of revolution, which is re-entrant, the path traced by the point of the revolving curve, nearest the axis, is known as the *circle of the gorge*, and the plane of this circle, as the *gorge plane.*

159. To assume a point upon a surface of revolution and to draw a tangent plane to the surface at the point.

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* This is sometimes called a spheroid; a prolate spheroid, if the revolution is about the major axis, an oblate spheroid if the revolution is about the minor axis.

† In discussing surfaces of revolution, let it be understood that the axis of the surface is taken parallel to one of the coordinate planes and perpendicular to the other, which is the most convenient method of representation for the purpose of analysis.

Analysis:—*The point will lie on a meridian curve of the surface, which can be drawn, and a tangent plane to the surface, at the point, will contain a tangent to the meridian curve, through the point. Therefore, draw the meridian plane through the point, and if it is not the principal meridian plane, revolve it, and the point about the axis, until it becomes the principal meridian plane. Its projection, upon one coordinate plane, will lie upon the contour of the surface. Revolve the point back again into its original position; the projection of the path, on one coordinate plane, will be a circle, that on the other will be a parallel to the G.L.*

The tangent plane contains, in addition to the tangent to the meridian curve, through the point, the tangent to the circular path of revolution of the point of tangency, which, in turn is parallel to one of the coordinate planes; and therefore, one trace of the tangent plane will be parallel to one projection of this line.

Construction:—Let the surface be given as an oblate spheriod, shown as in Fig. 77 with axis parallel to V and perpendicular to H. Either projection of the point can be assumed as a'. Its path of revoluton, into the principal meridian plane, is a horizontal line in V projection, and a' falls at a_1', and its H projection at a_1. Revolving the point back again to its originally assumed position, a_1 moves in the arc of a circle, with o as a center, to the position a, on a perpendicular through a'. a', as assumed, could stand for two positions of the point on the surface, the other being a_2. The tangent to the surface at A can be drawn, at the revolved position of A, the revolved V projection of its H trace being b_1'. The H trace, B, re-

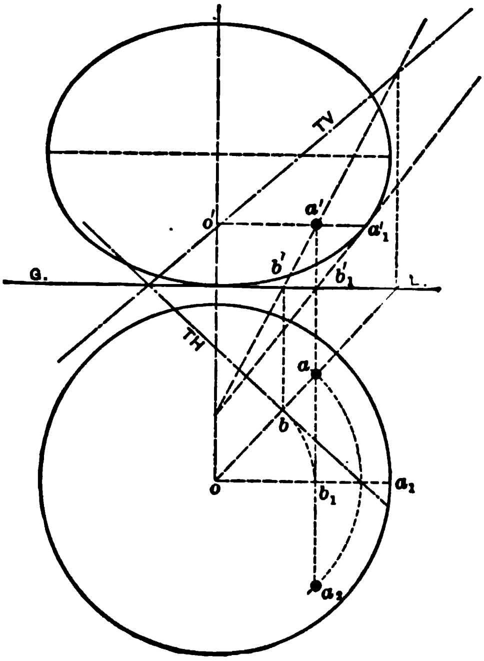

FIGURE 77.

volves back to the original position, and falls at *b'* in V projection; and *b* in H projection, and the H trace of the required plane, T, is tangent to the circle of the path of B, at *b*. The V trace is obtained in the usual manner, or it goes through the V trace of the tangent to the surface at A.

The tangent to the meridian curve, in revolving from the one position to the other, generates a portion of the surface of a right circular cone, whose apex is in the axis and its line of tangency with the surface may be considered its base.

160 THEOREM XV.

A plane which is tangent to a surface of revolution at a given point is perpendicular to the meridian plane through the point.

> **Proof—The tangent plane contains the tangent to the circle of revolution of the point. The tangent, being perpendicular to the radius of this circle, is also perpendicular to any line connecting the point of tangency with the axis of the surface, since the radius is perpendicular to this axis. But the radius and the axis lie in a plane, hence the tangent is perpendicular to this plane which is the meridian plane.**

161. To pass a plane tangent to a sphere, and through a given line outside.

Analysis—*The given line will be the intersection of the two possible planes tangent to the sphere, through the line. Each of these planes will be perpendicular to a radius of the surface through the point of tangency. The plane of these normals will be perpendicular to the required planes. Therefore, pass a plane through the center of the sphere, and perpendicular to the given line. From the point in which it is pierced by the line, draw a tangent to the great circle cut from the sphere by the plane. This line, and the given line, constitute one of the possible tangent planes.*

Construction:—Let the sphere be given, as in Fig. 78, and the line as AB. Pass a plane through the center of the sphere, and perpendicular to the line AB by using a horizontal of the plane through O. The line AB pierces this auxiliary plane in the point P. Revolving the center of the sphere O and the point P into H, P falls at p_1. One

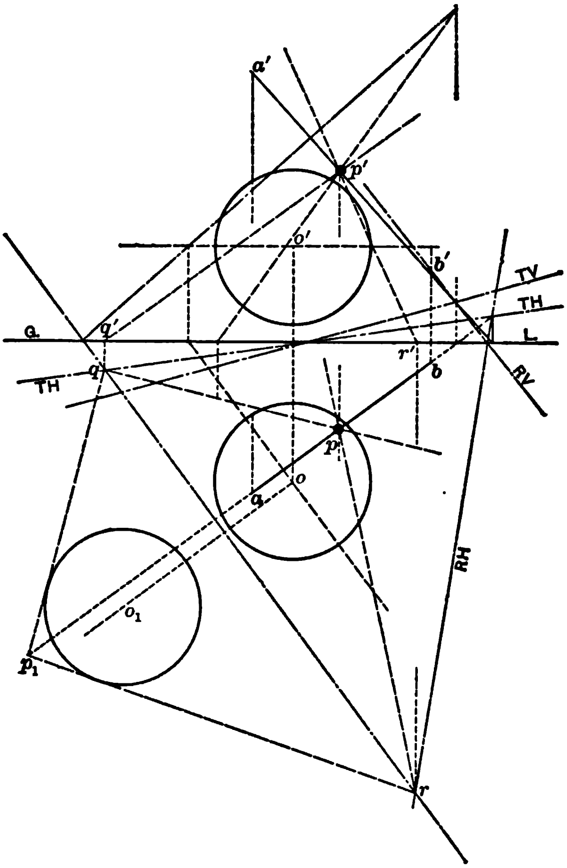

FIGURE 78.

tangent to the great circle cut from the sphere, by the auxiliary plane, is $p_1 r$, the other is $p_1 q$. Hence, the plane of AB and PR, in the one case, and AB and PQ in the other, determine the required tangent planes, R and T, respectively.

162. THEOREM XVI.

If two surfaces of revolution, having a common axis, are

tangent to each other, or intersect, it will be in the circumference of a circle, whose plane is perpendicular to the axis, and center in the axis.

> **Proof:**—If a plane is passed through any point of the curve of intersection, and the common axis of the surfaces, it will cut a meridian curve from each surface, intersecting each other in a point. If each meridian curve is revolved about the axis, it will generate the surface to which it belongs, while the point, common to both the meridian curves, will describe the circumference of a circle, common to both surfaces, with center in the axis, and plane perpendicular to the axis.

163. To find the curve of intersection of any surface of revolution by an oblique plane.

Analysis:—*Planes, passed through the axis of the surface, will cut meridian curves from it, and right lines from the secant plane, the points of intersection of which lie on the curve of intersection of the surface and the plane. Or again, planes perpendicular to the axis, will cut circles from the surface and lines from the secant plane.*

In construction, the latter are to be preferred because the circles are easily drawn, if the axis of the surface is perpendicular to a coordinate plane, and the lines cut from the secant plane will be horizontals or verticals of the plane.

Construction:—Let the surface be a torus form, a name derived from its use in architecture, a surface formed by the revolution of a circle about a line lying in its plane, but not passing through the center. It is shown in Fig. 79, and the secant plane as T.

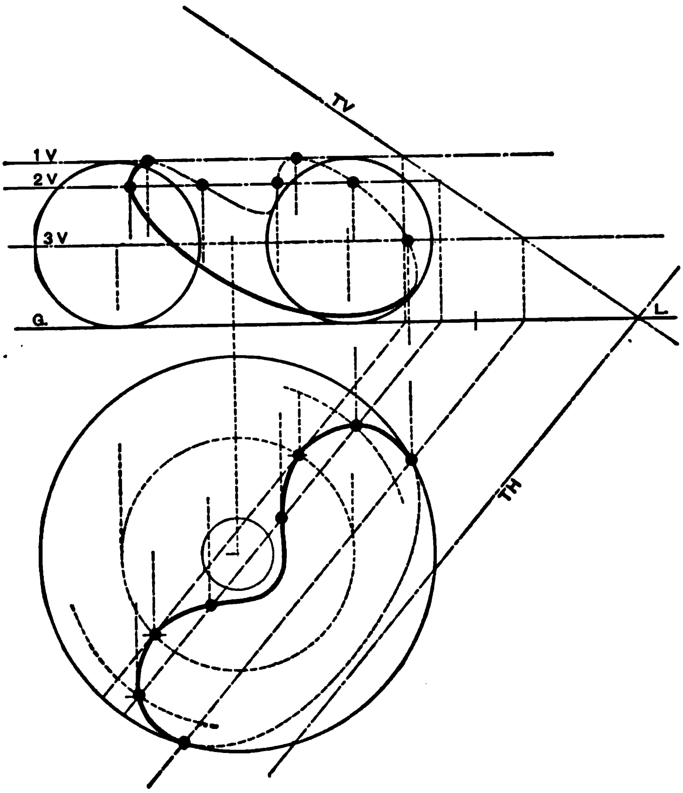

FIGURE 79.

Pass a series of secant planes perpendicular to V, and to the axis of the surface, as shown at 1, 2, 3, etc. Their V traces will be parallel to the G. L., and the horizontals cut, thereby, from the plane T will intersect the respective circles, cut from the surface, in points of the curve of intersection. The V traces of these lines are identical with the traces of the secant planes respectively.

164. The hyperboloid of one nappe, or hyperboloid of rev-

olution, as it is called, is a surface generated by a straight line, which moves so as to touch three windschief lines, which are equi-distant from a fourth, and make equal angles with a plane, perpendicular to the fourth. Or it may be considered as having two rectilinear directrices, and a right circular cone directer.

This surface is distinguished from the hyperbolic paraboloid by the conditions that the three windschief directrices shall be equally distant from a fourth, and shall make equal angles with a plane perpendicular to the fourth. .

Its construction, however, is best understood when made according to another statement of the law of generation, namely, a surface generated by a line revolving about another line, which it does not intersect.*

Let Fig. 80 represent a hyperboloid of revolution, constructed as follows: — Let the axis of revolution, OP, be perpendicular to H, and AB be a limited portion of the revolving line, shown here as parallel to V, and moving around OP, in the direction of the arrow. Its perpendicular distance from the axis is *ep*. Every point of the line AB describes a circle, whose center is in the axis, and whose plane is perpendicular to the axis; hence, when any point, as G, has revolved to a position G_1, in a plane with the axis OP, which is parallel to V, it will be a point in the contour of the surface; for the contour is the line of tangency of a cylinder of projection, and since the surface is one of revolution the contour is in a plane parallel to V. All points of the line AB will take, successively, positions in the principal meridian plane, in the revolution about the axis, and other points in the contour may thus be plotted.

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*A law shortly to be proved.

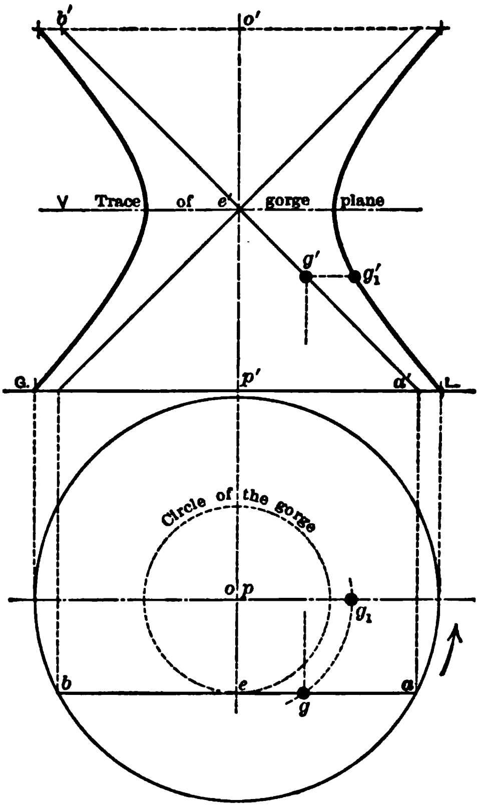

FIGURE 80.

This surface is symmetrical on its axis, so that for every point, as G, there will be one symmetrical with it upon the opposite side of the axis, and the V projection, in this case, will be symmetrical with respect to the line $o'p'$. For convenience, in the illustration chosen, AB is bisected at E, which describes the smallest circle of revolution of any point of the line; it is the *circle of the gorge*, and its plane is *the gorge plane*.

165. THEOREM XVII.

The hyperboloid of revolution of one nappe has two systems of generation, and every element of the one system intersects all those of the other system.

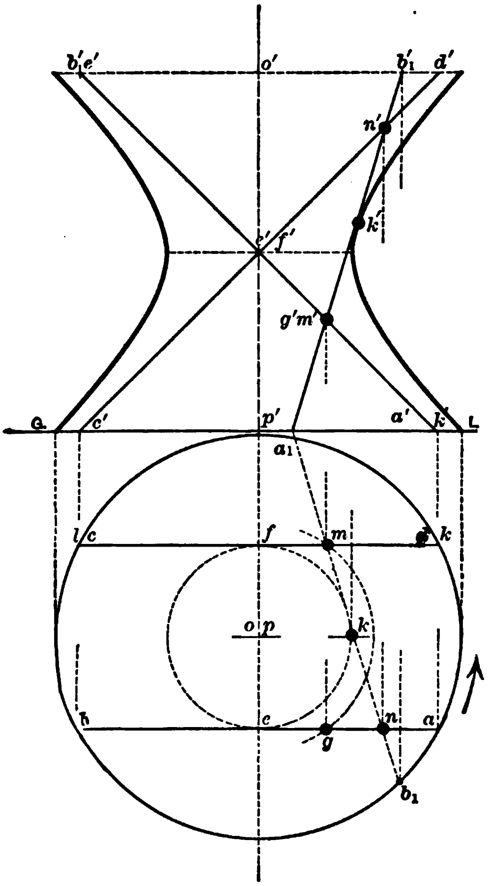

FIGURE 81.

Proof—If any point of an element AB as E in Fig. 83., revolve through an angle of 180°, it will occupy the position F, and the element, that of the position of CD, making the same angle, with the plane perpendicular to the axis, as AB does. If, now, through the point F, another line KL is drawn making the same angle with this plane, and lying in a plane parallel to V, it will, if revolved about the axis, generate the same surface, for, any plane passed perpendicular to the axis, will cut both these elements in points, respectively, G and M which are equally distant from the axis, and these each, in revolution, will describe the same circle perpendicular to the axis, and so for any other piercing points of the elements with any plane perpendicular to the axis; hence, the surfaces generated, by the elements, will be identical. And, since this is true, through any point of the surface, two right lines can be drawn, which are elements of the two systems of generation, respectively.

Since the point G, of the element AB, and the point M, of the element KL, generate the same circumference, it follows that, at some position of the element AB, the point G coincides with the point M. This position is shown by the dotted line A_1B_1, and so for any other point of AB. Hence, if either generatrix remain fixed, the other one, in revolution, would intersect it in all points successively.*

And, if any three generatrices of one system, are taken as directrices, and a generatrix of the other system moves, so as to touch the three, it will generate the surface, hence the definition.

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*This property is one characteristic of warped surfaces in general having double systems of generation.

166. THEOREM XVIII.

The meridian curve of a hyperboloid of revolution of one nappe is a hyperbola.

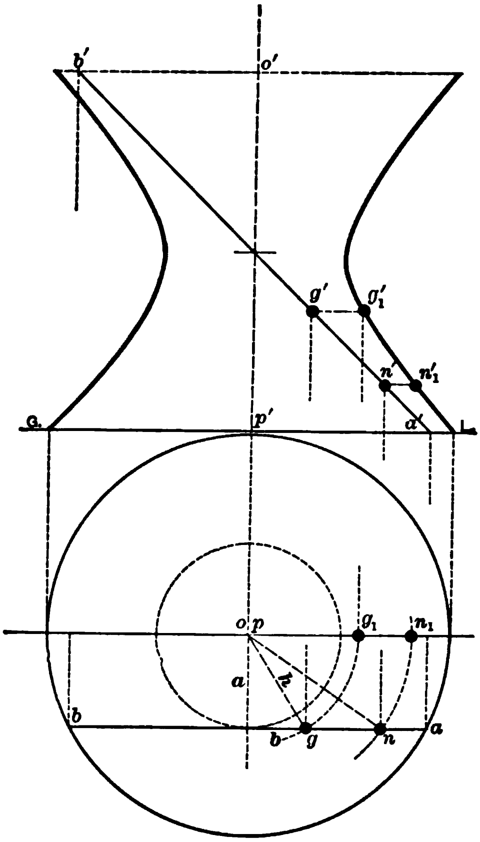


FIGURE 88.

Proof:—Let Fig. 83 be a hyperboloid with G and N, any two points upon it. The horizontal distance of n_1' from n' is less than that of g_1' from g'. Still further removed from G, the corresponding projection of the distance of any point, on the meridian curve, from the projection of the generatrix, when parallel to one coordinate plane, is less. This distance, in any case, as $g_1'g'$, is the difference between the radial distance of the point from the axis, as $g\ o$, and its distance in H projection from the point of tangency of the element AB, to the circle of the gorge. The first distance is the hypotenuse of a right angled triangle of which the second distance is the base, and the distance of the point of tangency, in H projection, from the axis is the altitude. The hypotenuse, and the base vary, for every point on the element, while the altitude remains constant. Let the hypotenuse be represented by h, and the base by b, and the altitude by a. Then the conditions may be expressed algebraically, as

$$a^2 = h^2 - b^2$$

$$\text{or } a^2 = (h + b)\ (h - b)$$

whence

$$(h - b) = \frac{a^2}{h + b}$$

The left hand member diminishes in value as the denominator of the right hand member increases, and becomes equal to zero when the denominator becomes infinite. Therefore, the element AB is an asymptote to the curve. Again, when b = zero, $h^2 = a^2$, and the radius of the circle of the gorge is equal to a, the nearest point of the moving element to the axis.

Referring also to Fig. 80, and remembering that the line through $a\ b$ may stand for the H projection of an element of each system, assume any other position of AB as A_1B_1. It will cut one element of the other system in a point M., and a second element in a point N,

on the opposite side of the circle of the gorge. Now, since this element has but one point in common with the meridian curve, the vertical projection of A_1B_1 will be tangent to this curve at K. But $n\ k = k\ m$, then $m'\ k' = k'\ n'$, that is, the tangent to the meridian curve is bisected at the point of contact, a well known property of the hyperbola. Hence, the meridian curve is a hyperbola; in the figure, it has a horizontal transverse axis and a vertical conjugate axis.

The surface could be generated, therefore, by revolution of a hyperbola about its conjugate axis.

If the circle of the gorge diminishes in diameter to zero, the generatrices will go through its center and the surface becomes a cone.

If the angle of the generatrices with the plane directer diminishes to zero the surface become a plane.

167. To assume a point on the surface of a hyperboloid of revolution of one nappe, and to draw a plane tangent to the surface at the point.

Analysis:—*The elements of both systems of generation, which pass through the assumed point, will each be tangent to the projection of the circle of the gorge, on a plane perpendicular to the axis of the surface, and intersect any assumed bases of the surface in points, which can be found. And the tangent plane will contain the elements of the surface—one of each system—passing through the point of contact, will be tangent to the surface at a point, and cut the surface in two straight lines.*

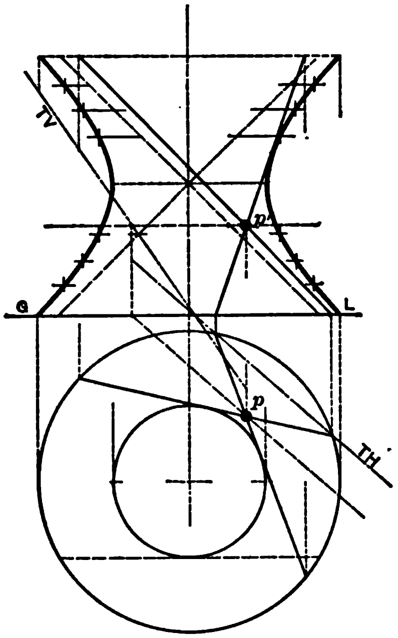


FIGURE 83.

Construction:—Let Fig. 83 be a hyperboloid of revolution.

First, assume P, in H projection, in which case either or both elements can be drawn through it, tangent to the circle of the gorge, and cutting the base in the circle of contact with H, from which their V projections may be obtained. Or the V projection of P may be assumed. In this case, revolve the point about the axis until it falls into the principal meridian plane, and after its H projection has been obtained in this position, then revolve it to its original position and obtain the true H projection.

The tangent plane contains the elements, one of each system, passing through the point of contact, and its traces pass through the traces of these elements. TV and TH are the traces as shown.

168. To find the curve of intersection of the hyperboloid of revolution with any plane.

Analysis:—*The most convenient method of finding the curve of intersection, is to pass a series of auxiliary secant planes perpendicular to the axis of the surface, cutting circles from the hyperboloid, and right lines from the given plane, the respective points of intersection, of which, give points upon the curve of intersection of the two.*

CHAPTER VIII.

INTERSECTIONS OF SURFACES.

169. Two surfaces will intersect each other in curves or straight lines, according to the character of the surfaces. And the method of ascertaining the form of the curve, will depend also upon the properties of the surfaces considered; no general procedure can be laid down, except the following: *Pass a series of secant planes to cut from both surfaces, the simplest lines that can be drawn, the intersections of the curves cut from both surfaces by the secant planes, respectively, will give points in the curve of intersection of the surfaces.*

The subject of intersections is best investigated by working out a series of problems involving different kinds of surfaces. The intersection of the simpler single curved surfaces, and surfaces of revolution are of particular interest to the student, and furthermore, intersections, together with the development of the surfaces intersected, form a class of problems of most frequent occurrence in practical descriptive geometry, and deserve to be dwelt upon.

As the student has by this time become accustomed to the fundamental processes, it is proposed to curtail the analyses, somewhat, and to cover only the special features involved in each new problem.

170. To find the intersection of two bodies bounded each by plane faces.

General Analysis: — *The curve of intersection will connect the points of intersection of the various edges of the one form with the surfaces of the other form. Hence, (1), find the various lines of intersection, of the planes of the one form, with those of the other, respectively, or, (2), find the traces of the edges of the one form with the faces of the other, the latter extended if necessary to contain the traces.*

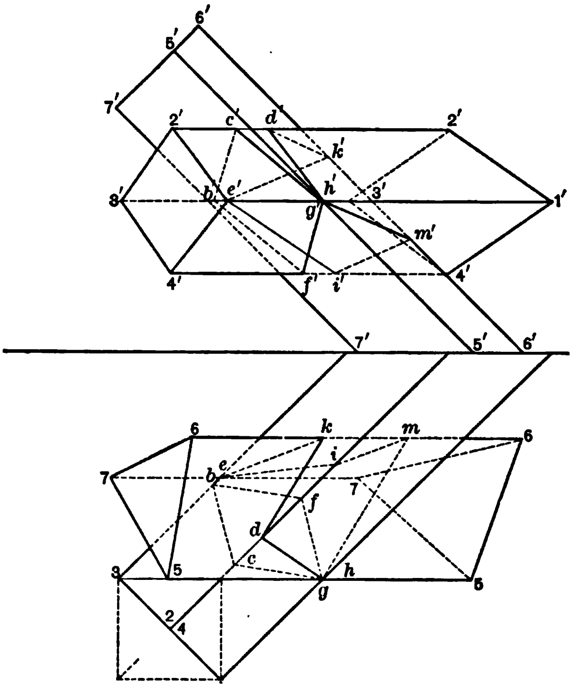


FIGURE 84.

Construction: — Let the two forms be two prisms, square and triangular, as in Figure 84. Starting with any edge of one of the prisms, as 1, of the square prism, see if it intersects the surfaces of the triangular prism, by using the H projecting plane of the edge, and finding its line of intersection with the faces of the triangular prism, and so on, systematically, each edge in turn, and then find, similarly, the piercing points of the edges of the triangular with the square prism, wherever the intersection has not been determined.

171. One form will completely penetrate the other when two parallel, tangent and diametrically opposite, planes to the one form cut lines out of the second form. This construction is very useful sometimes, in determining the sphere within which to work with least expenditure of effort. Two sets of tangent planes, one set to each form, will determine at once the limits of the curve, or curves, of intersection.

To make the statement for complete or partial penetration general, for all surfaces, it may be said that one completely penetrates the other when two tangent, and opposite, planes to the one form, *and constructed in accordance with the limitations imposed by the character of the surface*, cut the second form.

If one completely penetrates the other, there will be two distinct curves of intersection, if not, the curve of intersection will be single and continuous.

172. To find the lines of intersection of two pyramids.

Analysis:—*Find the piercing points, of the edges of one, with the surfaces of the other, by means of auxiliary planes, passed through the edges of each, and, when possible, pass them through the apexes of both pyramids; whence, the* *common line of intersection, of the several secant planes, is the line connecting the apexes of the two pyramids, and the lines cut from both pyramids are elements of them respectively.*

173. To find the curve of intersection of two cylinders.

Analysis:—*If the planes of the bases of the two cylinders are parallel, choose the secant planes parallel to the bases. Similar curves to the bases will be cut from each, and their respective intersections give points on the curve desired. If the bases are not in parallel planes, pass secant planes through the elements of one, which are parallel to those of the other, hence, cutting elements, if at all, out of both.*

Construction:—Let two cylinders be given, as in Fig. 85, having axes AB and CD respectively. Since the second cylinder has its axis parallel to the G. L., any auxiliary planes, passed parallel to its elements, will be parallel to the G.L. Hence, to find the intersections of such a series of auxiliary planes with both cylinders, project the cylinders upon and end plane as shown. The respective intersections of the contour of both cylinders, with any one auxiliary plane, are the piercing points of elements of the cylinders, with their respective bases, and these can be transferred to the coordinate planes. One such auxiliary plane is shown, cutting the elements EF and IJ, respecively, out of both cylinders.

Note. In the practical application of descriptive geometry, in such problems as these, it is found convenient to choose one coordinate plane parallel to the axes of both solids, so that the elements cut out will be projected their true length on that coordinate plane, since the main object

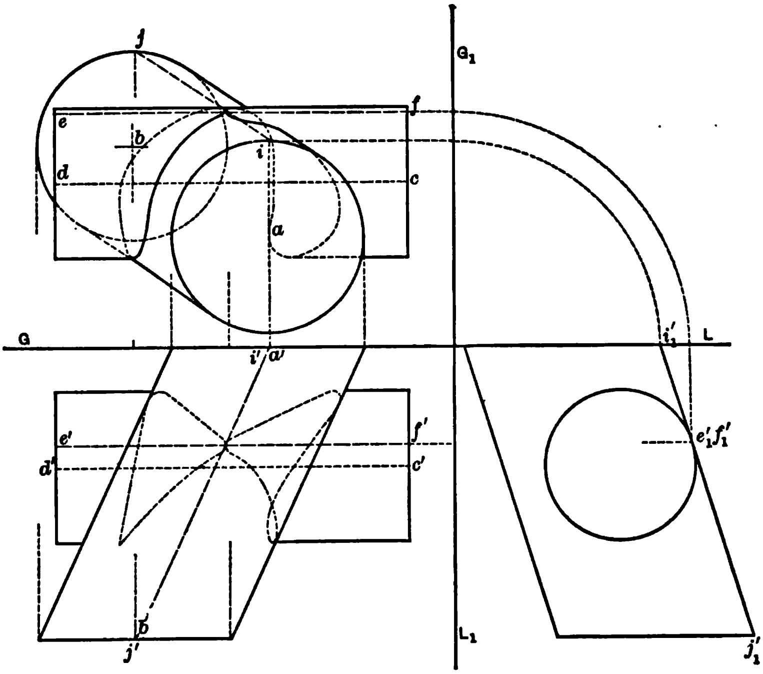


FIGURE 85.

of ascertaining the intersection is that the forms may be developed, such for example as two intersecting pipes.

174. To find the curve of intersection of two cones.

Analysis:—*Unless the bases are circular and in parallel planes, pass a series of secant planes to cut elements from both cones. These planes will intersect each other in a line through the apexes of both cones, and their traces, moreover, will go through the traces of this line. The respective intersections of the elements, cut from the cones, will be points in the curve of intersection sought.*

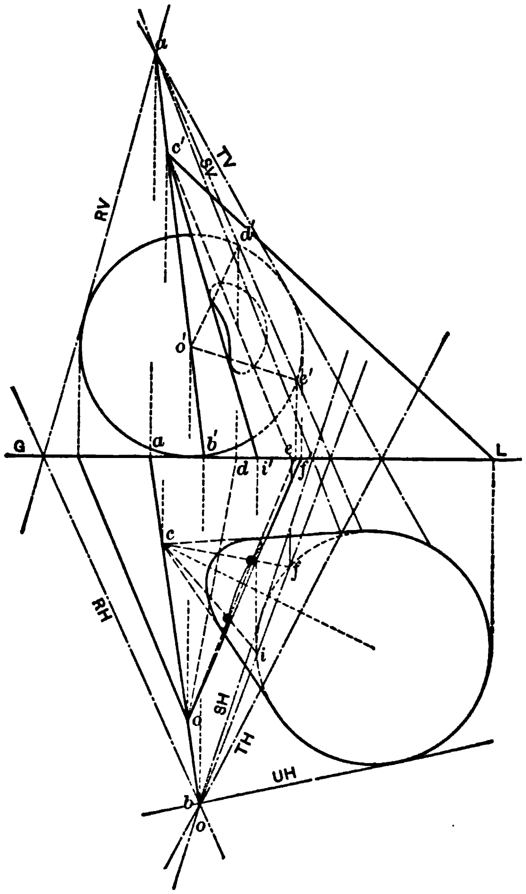


FIGURE 86.

Construction:—Let two cones be given, as in Figure 86. The circular base of one is in V, with apex at O, and the circular base of the other is in H, with apex at C. The line OC, connecting the apexes, has its V and H traces in A and B, respectively, and the secant planes used will contain these traces. Planes T and R, drawn tangent to the first mentioned cone, show that T cuts elements out of the second, but R does not, likewise planes S and U, drawn tangent to the second show that S cuts elements out of the first, but U does not; hence, neither cone completely penetrates the other. The construction of points by use of another auxiliary plane is shown.

If both bases are in the same coordinate plane, only one trace of a secant plane is necessary to determine whether that plane cuts both cones. Its corresponding trace may be found with a horizontal, or vertical, of the plane, or it may not be needed in construction.

If both cones have the same apex, either trace of any secant plane can be assumed at will, the corresponding trace being determined by the fact that the apex is a point in the plane.

If the line connecting the apexes of the cones, is parallel to their bases, the traces of the auxiliary planes, with those bases, will be parallel lines.

175. To find the curve of intersection of a cone and a cylinder.

Analysis:—*The secant planes, to cut elements out of each, must pass through the apex of the cone, and be parallel to the elements of the cylinder; hence, draw a parallel line through the apex of the cone, as the common line of intersection of the secant planes.*

Construction:—Let the cone and cylinder be given as in Figure 87. Draw a line through the apex of the cone, as the line of intersection of the secant planes. In this case, it is perpendicular to H. The secant planes T, R, S, etc., are, therefore, perpendicular to H, and cut both surfaces in straight lines. The H traces can be assumed at will, to pass through the H trace of the auxiliary line,

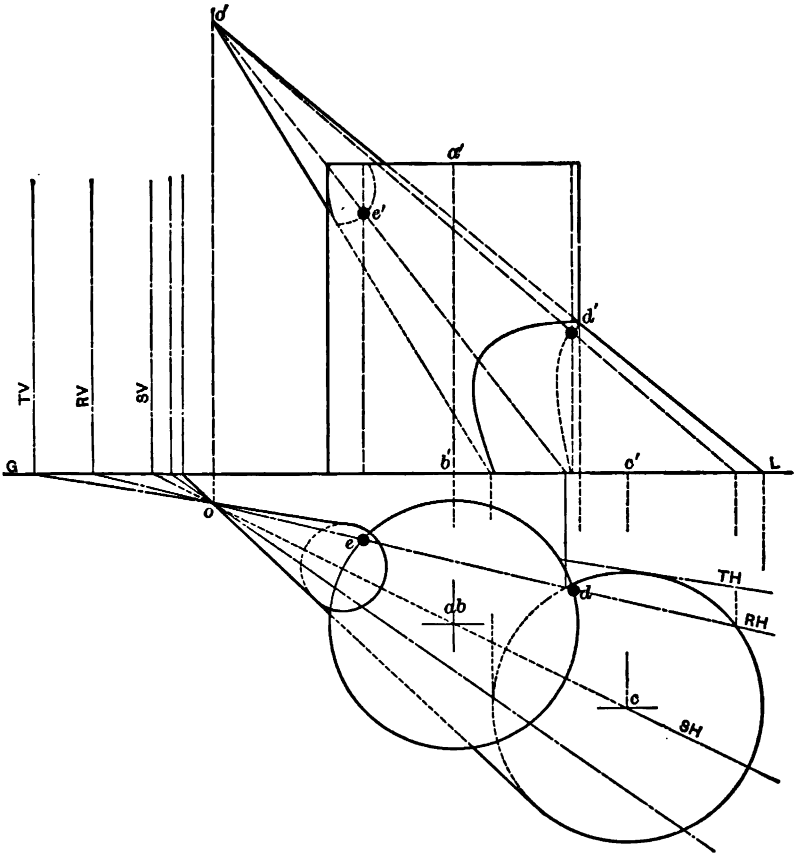


FIGURE 87.

through the apex of the cone. The points of intersection, D and E, of the elements cut out of each, by one auxiliary plane R, are shown.

Where the bases of cone and cylinder are in the same plane, the traces of the secant planes, with that plane, can be assumed at will to go through the trace of an auxiliary line, through the apex of the cone, and parallel to the elements of the cylinder. If this auxiliary line is parallel to the plane of the bases, the traces of the secant planes, on that plane, are parallel.

If the bases of cone and cylinder are in or parallel to different coordinate planes, then either of the traces of the auxiliary planes may be assumed at will, to go through the similar trace of an auxiliary line, through the apex of the cone, and parallel to the elements of the cylinder.

The proper traces to assume, of the auxiliary planes, are those with the coordinate plane in which the bases respectively lie, for the traces show, at once, the elements cut from that solid whose base lies in that plane.

176. To find the curve of intersection of a cylinder and a convolute.

General Analysis:—*If the base of the cylinder lies in or parallel to the plane of section of the convolute, used to represent part of the contour of the latter, then secant planes, parallel to the base of the cylinder, will probably give the simplest construction, although cutting curves out of both. If the base of the cylinder is not so related to the convolute, then, through any point of an element of the latter, draw an auxiliary line parallel to the elements of the cylinder; the plane of these two lines determines the direction of one auxiliary plane.*

177. To find the curve of intersection of a cone and a convolute.

General Analysis:—*If the base of the cone lies in, or parallel to the plane of section of the convolute, used to represent part of the contour of the latter, then secant planes, parallel to the base of the cone, will give the simplest construction, although cutting curves out of both.*

If the base of the cone is not so related to the convolute, the general solution involves the cutting of a series of differing curves from the convolute, while cutting curves or elements from the cone.

178. Two intersecting surfaces of revolution may come under one of three groups. 1. When they have a common axis. 2. When the axes are in the same plane. 3. When the axes do not intersect.

If two intersecting, or tangent, surfaces of revolution have a common axis, they will touch each other on a circle or circles; for, having a common axis, any point, common to the two, must lie on a meridian curve, and, in each case, if revolved, will generate the surface, every point traveling in the path of a circle, whoes center is in a plane perpendicular to the axis.

179. To find the intersection of two surfaces of revolution having a common axis.

Analysis:—*If the axis is taken parallel to one coordinate plane, and perpendicular to the other, then secant planes, parallel to the latter, will cut both surfaces in circles, which intersect in points desired. If the axis is not so placed, a change of coordinate plane or planes would be most desirable.*

180. To find the curve of intersection of two surfaces of revolution whose axes are in the same plane.

Analysis:—*If the axes are parallel, pass a series of auxiliary planes perpendicular to the axes, the curves cut from each surface will be circles.*

If the axes are not parallel, the convenient method is to intersect both surfaces with a series of spheres, of different radii, and with centers at the point of intersection of the axes. These spheres will cut circles, respectively, from both surfaces. If the plane of the axes is taken parallel to a coordinate plane, the projection of these circles on this coordinate plane will be limited and intersecting straight lines, the points of intersection of which, respectively, are the common chords of the circles cut from each surface, one point each, as a common chord to the two circles cut from the surfaces by any one sphere.

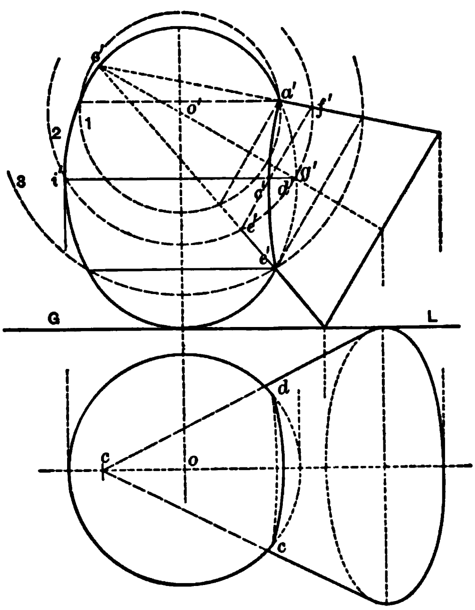


FIGURE 88.

Construction:—Let the two surfaces be an ellipsoid of revolution, or prolate spheroid, and a right elliptical cone, whose apex C, is on the surface of the spheroid. See Figure 88. The center for a series of auxiliary circles is at

O, the intersection of the axes of the surfaces. A sphere, such as 1, and another such as 3, since they go through the points of intersection of the contours of the solids, and since the plane of the axes is parallel to V, will cut out chords of zero length, i.e., be the piercing points of those elements of the cone which are parallel to V. Sphere 2 will cut out a circle from each intersecting in a common chord CD, the circle cut from the spheroid is vertically projected in the line $i'g'$ and the circle cut from the cone, is vertically projected in the line $e'f'$. In like manner other points are ascertained.

181. To find the curve of intersection of two surfaces of revolution who axis are not in the same plane.

Analysis:—*Pass secant planes to cut the simplest series of lines out of each surface, circles out of one, for example, by planes perpendicular to the axis, and curves out of the other.*

No general analysis leading to a simple solution under these conditions is possible.

182. To find the curve of intersection of a single curved surface and a surface of revolution.

Analysis:—*Pass a series of secant planes normal to the axis of the surface of revolution, to cut circles from it and circles, ellipses, or other curves from the other surface.*

If the single curved surface is a cone, or cylinder, it is, in general, possible to get fairly simple curves of intersection. If the surface is a sphere, also, the curves of intersection will be circles. If it is a convolute surface, the curves cut out will depend upon the position of the axis of the convolute relative to that of the surface of revolution.

APPENDIX.

Practical projection is based upon descriptive geometry, but the short cuts, in common use, are, many of them, entirely at variance with it. They are correct in their way, but the practical and the theoretical should not be confused. For example, the real difference between first and third angle projection has not always been followed by draftsmen, the two have been mixed. In the first angle, an object is projected upon the coordinate planes from a center of projection, which is on the same side of the plane as the object; in the third angle, the center of projection is on the opposite side of the plane from the object. This, in the case of the first angle, results in an end view of an object being placed upon the opposite side from the one the view is supposed to represent, while, in the third angle, the end view comes next to the side which it is supposed to represent. Further than this, in the first angle, the revolution of the end view is away from the end it represents, and in the third angle it is toward it, theoretically, while in practical projection, for both angles, the revolution is away from the end it represents, thus, theoretically, dividing the end plane, on its intersection with V, and folding the front and rear parts on each other like the leaves of a book. If these distinctions are kept in mind, the student will not be likely to mix the first and third angles in his practical work. Since practical projection is but a language constructed to give expression to ideas, it should be allowed license in its mode of expression, to best suit its purposes, as long as there is consistency, and a common understanding and approval of what is meant.

Practical projection ignores the ground lines between any coordinate planes of projection, and for the very good reason that they in no way affect the projections of forms or the relations of the projections to each other except as to distance apart; and, because ground lines are ignored, in various other ways, detail views are arbitrarily placed where they will be most explanatory. As a concrete illustration of this, the section of a spoke of a wheel may be revolved about the median line of the section, where taken, and shown as limited by the outlines of the spoke. Again, it may be said, that such arbitrary constructions are permissible if well understood and accepted by custom.

A very large share of the practical problems involving descriptive geometry are those of ascertaining the direction and shape of sections of forms, and of the development of the forms. Not all peripheries of solids can be developed, in the true sense of the word, while, in reality, all forms are developed in practice. The only forms that can be developed, in conformity with the principles of descriptive geometry, are those having straight line elements and intersecting, at least consecutively, two and two. No double curved or warped surfaces are developable.

In practical work these latter surfaces frequently must be shaped up, notably in the sheet metal worker's business, and interesting devices are resorted to. For double curved surfaces there are two general methods: 1. The zone. 2. The gusset. In the zone method, the solid is divided into zones, by planes perpendicular to a principal axis, if there be any, and the zones again broken up into two or more parts, to get smaller pieces, more closely approximating the real form. The gusset method is best illustrated by reference to the periphery of an orange when divided into its natural lobes, or to a sphere, divided by meridians into the necessary number of slices, closely approximating the true form. In the sheet metal worker's business, the material is sometimes punched out or stretched to make it fill out the remaining difference between the true and the constructed forms.

Other forms are developed by a system known as triangulation. The surface is divided up into triangles having their corresponding bases oppositely directed. In the development, each triangle is revolved about its edge of contact with an adjoining one, and the bases of the alternate triangles form a continuous development of the perimeter of a portion of the form; opposite to these, the bases of another set of alternate triangles go to make up another portion of the perimeter. The frustrum of an oblique cone is a surface which might be dealt with in this way, but the method can also be applied to other surfaces.

EXERCISES.

FOLLOWING SECTION 24, PAGE 17.

1. Draw the projections of the following points: A $= 0, 1\frac{3}{8}, \frac{3}{8}$; B $= \frac{3}{4}, 1\frac{3}{4}, -\frac{1}{8}$; C $= 1\frac{1}{4}, -1, -\frac{1}{2}$; D $= 2\frac{1}{8}, -1\frac{3}{8}, \frac{1}{2}$; E $= 3\frac{1}{4}, -1\frac{3}{8}, 0$. Project them upon a new V_1 plane whose $G_1.L_1.$ cuts the G.L. at $3\frac{1}{4}, 0, 0$, and makes an angle of 30° with it downward towards the right.

2. Draw the projections of the following lines: A B has A $= 0, \frac{7}{16}, -\frac{7}{16}$; B $= 1\frac{1}{8}, 1\frac{1}{16}, -1\frac{1}{16}$. C D has C $= 2, \frac{13}{16}, \frac{3}{4}$; the line pierces the G.L. and d' is at $3\frac{9}{16}, -\frac{5}{8}, 0$. Project both of these lines upon a new V_1 plane whose $G_1.L_1.$ is parallel to the H projection of C D and cuts the G.L. at $1\frac{7}{8}, 0, 0$.

FOLLOWING SECTION 26, PAGE 19.

3. Given two points A $= 0, 1\frac{1}{8}, \frac{3}{8}$, and B $= 1\frac{3}{4}, -1\frac{1}{4} -1\frac{1}{2}$; draw the projections of the line joining them.

4. Given two points, A $= 0, -1\frac{1}{4}, -1$, and B $= 2, \frac{3}{4}, 1\frac{1}{8}$; draw the projections of the line joining them.

5. A B, C D and E F are three lines. A $= 0, 0, 0$, and B $= 1\frac{1}{8}, \frac{13}{16}, 1\frac{3}{16}$; C $= 1\frac{1}{2}, -\frac{3}{8}, -\frac{3}{8}$, and D $= 2\frac{3}{8}, -\frac{3}{8}, -1$; E $= 3, -1\frac{1}{8}, -\frac{3}{8}$, and F $= 4, 0, -\frac{3}{8}$. What are the relations of these lines to the coordinate planes? the angles CD and EF with them?

6. A B and C D are two lines. A $= 0, 2\frac{1}{8}, 1\frac{3}{16}$, and B $= 1\frac{1}{4}, 1, \frac{7}{16}$; C $= \frac{3}{4}, 1, \frac{5}{8}$, and D $= 1\frac{3}{4}, 1\frac{3}{8}, 1\frac{1}{2}$. Draw their projections.

7. A B is a line with A $= 0, \frac{3}{8}, 1\frac{1}{8}$, and B $= 1\frac{3}{4}, 1\frac{1}{2}, \frac{1}{8}$. C and D are points on the line; C is in a vertical plane $\frac{1}{2}$ inch to the right of A, and D lies in a horizontal plane $\frac{3}{8}$ inch below B. Locate both projections of the points.

8. A B is a line, with A $= 0, -1\frac{1}{4}, -1\frac{1}{4}$, and B $= 1\frac{1}{2} -\frac{1}{4}, -\frac{7}{8}$. C is at $1\frac{3}{8}, -1, -1\frac{5}{8}$. Through C, draw any line to intersect A B.

9. A B is a line, with A $= 0, 1\frac{7}{8}, -\frac{5}{8}$, and B $= 1\frac{1}{2}, 1\frac{3}{4}, -\frac{7}{8}$. C is the middle point of A B. D is a point at $\frac{1}{4}, \frac{1}{2}, \frac{5}{8}$. Through D, draw a line to intersect the line A B, in the point C.

10. A B is a line, with A $= 0, \frac{1}{2}, 1\frac{1}{2}$, and B $= 1\frac{5}{8}, 1\frac{1}{4}, \frac{1}{4}$. C is at $1\frac{1}{4}, \frac{1}{2}, 1\frac{5}{8}$. From C, as one extremity, draw a line so that its middle point shall lie on the line A B.

11. Locate the projection, on a right hand end plane, of a line whose one end is $\frac{1}{2}$ inch from V, and $\frac{3}{4}$ inch from H, in the first angle, and whose other end is $\frac{1}{4}$ inch from H, and $\frac{3}{4}$ inch from V, in the third angle. The ends of the line are $1\frac{1}{2}$ inches apart horizontally. Draw the projections of the line in either of its two possible positions upon the H and V planes.

12. Draw the projections upon the H, V and V_E planes of the two lines joining the points AB and CD. A $= 0, \frac{1}{2}, \frac{3}{8}$, and B $= 2, 1\frac{1}{4}, 1$; C $= \frac{1}{4}, \frac{1}{2}, -1\frac{3}{8}$, and D $= 1\frac{5}{8}, -1, \frac{5}{8}$. Let $G_E.L_E$. cut the G.L. at $3\frac{3}{4}, 0, 0$.

13. Draw the projections, upon the H, V and V_E planes, of the two lines joining the points A B and C D. A $= 0, 1\frac{1}{2}, -\frac{1}{2}$, and B $= 0, -\frac{5}{8}, 1\frac{1}{2}$; C $= \frac{1}{2}, -1\frac{3}{4}, -1$, and D $= 2, 0, 0$; $G_E.L_E$. cuts the G.L. at 3, 0, 0.

14. Given the line A B, with A $= 0, 1\frac{1}{4}, \frac{3}{8}$, and B $= 1\frac{3}{4}, \frac{1}{4}, 1\frac{3}{4}$; draw the projecting planes of the line.

15. A point A $= 0, \frac{5}{8}, \frac{1}{4}$ and a point B $= 0, 1\frac{1}{2}, 1$; find the projections of a line C D, 2 inches long and parallel to the line A B, and distance $\frac{5}{8}$ inch from it, in a vertical plane through the line A B.

16. Draw the lines A B and C D which intersect each other in a point O, at $0, \frac{3}{4}, -1\frac{1}{2}$. A B is parallel to V and oblique to H, and C D is parallel to H and oblique to V. Coordinate the extremities of the limited line used.

17. Given the line A B C, B being the middle point in it. A

$= 0, 1\frac{1}{2}, \frac{3}{8}$, and C $= 0, \frac{1}{2}, 1\frac{1}{4}$. Draw a line through B parallel to a line E F, in which E $= \frac{3}{4}, \frac{1}{4}, \frac{1}{2}$, and F $= 1\frac{3}{4}, \frac{5}{8}, \frac{3}{4}$.

18. Given a line A B; A $= 0, \frac{3}{8}, \frac{5}{8}$, and B $= 1\frac{1}{8}, 1\frac{3}{8}, 1$. Draw limited lines parallel to it lying in each of the four dihedral angles.

19. Prove that the lines of problem 12 do not intersect.

TO FOLLOW SECTION 32, PAGE 24.

20. Find the traces of the lines A B and C D. A $= 0, 1\frac{1}{2}, -\frac{5}{8}$, and B $= 1\frac{1}{2}, -\frac{3}{8}, 1\frac{1}{4}$; C $= 1\frac{3}{8}, 1\frac{3}{4}, \frac{1}{4}$, and D $= 3\frac{3}{8}, \frac{3}{8}, 1\frac{1}{4}$.

21. Find the traces of the line A B. A $= 0, -\frac{3}{8}, -1\frac{3}{4}$; B $= 2, \frac{7}{16}, \frac{7}{16}$.

22. Find the traces of the line M N. M $= 0, 1\frac{1}{4}, 1\frac{1}{4}$; N $= 2\frac{3}{8}, 0, 0$. Also find its traces with an end plane $G_E.L_E$. cutting G.L. at $1\frac{3}{8}, 0, 0$, showing same as revolved into V about $G_E.L_E$.

23. A B is a line with A $= 0, 1\frac{1}{8}, -1\frac{1}{8}$; B $= 1\frac{7}{8}, \frac{1}{4}, -\frac{1}{4}$. Locate its traces with V and H and also with an end plane whose $G_E.L_E$. cuts G.L. at 1, 0, 0, revolving the same into the V plane.

24. Find the traces of the broken line connecting successively the points. A $= 0, -\frac{3}{4}, -\frac{7}{16}$; B $= 1\frac{1}{8}, \frac{15}{16}, -\frac{1}{8}$; C $= 2\frac{3}{4}, \frac{3}{8}, 1\frac{3}{8}$; D $= 1\frac{13}{16}, 1\frac{3}{8}, -1\frac{3}{8}$.

TO FOLLOW SECTION 33, PAGE 25.

25. Draw a line A B, with its V trace at $0, \frac{3}{4}, 0$, which passes through the 1st and 2nd angles only.

26. Draw a line A B with its V trace at $0, \frac{7}{8}, 0$, which passes through the 2nd, 1st, and 4th angles.

27. Given a line A B with A $= 0, \frac{1}{2}, -\frac{1}{2}$; B $= 1, 1\frac{1}{4}, -1\frac{1}{4}$. What is the position of the line with respect to H and V; where are its traces?

28. Given a line A B, with A $= 0, 1\frac{3}{8}, -\frac{1}{2}$; B $= 1\frac{1}{2}, \frac{1}{2}, -1\frac{3}{8}$. Find its traces and also its traces with an end plane whose $G_E.L_E$

passes through the point of intersection of the V and H projections of A B.

29. The V trace of a finite line A B is 1½ inches below H and its H trace is 1½ inches back of V. Draw the projections of such a line and find its traces with an end plane whose $G_E.L_E.$ cuts through the middle of the line.

TO FOLLOW SECTION 38, PAGE 32.

30. A B is a line, with A = 0, ½, ¼; B = 1½, 1½, 1¼. Find the true length of the line, by revolving it about the horizontal projecting line of the point B until it is parallel to V. Show, by appropriate notation, the H and V projections of the arc of revolution of the point A.

31. A B is a line, with A = 0, ⅜, 1¼; B = 1¼, 1¾, ⅜. Find the true length of the line by revolving it about the vertical projecting line of the point B until it is parallel to H. Show, by appropriate notation, the H and V projections of the arc of revolution of the point A.

32. A B is a line, with A = 0, 1⅛, 0; B = 1½,.0, ¾. Find the true length of the line, by revolving it about the H trace of its H projecting plane into H. Also, find its length, by revolving it about the V trace of its V projecting plane into V.

33. AB is a line, with A = 0, 1⅜, ¾; B = 1½, ¾, $\frac{3}{16}$. Find the true length of the line, by projecting it upon a supplementary V_1 plane, taken in any convenient place, parallel to AB.

34. A line goes through the points A and B with A = 0, ½, — ½, B = 1, 1¼, — 1¼. Locate any two points, O and P, on it which are 1¼ inches apart.

35. Given the four points, A, B, C, and D. A = 0, ½, ⅝; B = 1¾, 1¼, 1½; C = $\frac{7}{16}$, 1¾, 1¾; D = 2, 0, ¼. Find the distances between the extremities of the two lines AB and CD. Also, find the distances between P, on AB, ⅝ inch from A, and Q, on CD, ½ inch from C.

36. A, the V trace of a line AB, is at 0, — 2⅛, 0; B the H trace of the line is at $2\frac{3}{8}, 0, -2\frac{1}{4}$; C, the H trace of another line intersecting AB, is at $\frac{7}{16}, 0, -1\frac{3}{4}$; D, the point of intersection has its $z = 1\frac{1}{8}$. Draw the lines AB and CD, and find the V trace of CD. Through D, draw a line whose V trace has $z = \frac{5}{8}$, and is $\frac{7}{8}$ inch below H.

37. $A = 0, \frac{3}{8}, \frac{1}{4}$; $B = \frac{7}{8}, 1\frac{5}{8}, 1\frac{1}{2}$. Find the true length of the line by the following methods: (a) by revolving parallel to V; (b), by revolving parallel to H; (c), by revolving about its H projection into H; (d), by revolving about its V projection into V; (e), by projecting upon a new V_1 plane parallel to the line, using a $G_1. L_1$ to cut the G.L. at $\frac{5}{8}, 0, 0$.

38. Find the true length of the line AB with $A = 0, 1\frac{1}{4}, -1\frac{1}{4}$; $B = 1\frac{1}{2}, 0, 0$.

TO FOLLOW SECTION 46, PAGE 38.

Where TO is given it means the point in which the plane T cuts the G.L.

Point 1 stands for a point of a plane in V, 2, for a point of the plane in H, i. e., points on the traces.

39. T is a plane with TO at 0, 0, 0. TH makes an angle of 30° with the G.L., and TV makes an angle at 60° with the G.L. both towards the right. A, B, C and D are four points in the plane T, and their vertical projections are as follows: $a' = 1, \frac{3}{4}, 0$; $b' = -\frac{1}{2}, -\frac{5}{8}, 0$; $c' = \frac{1}{2}, -1\frac{1}{4}, 0$; $d' = 0, 1\frac{1}{4}, 0$. Find the H projections of the points by using a horizontal of the plane, in each case.

40. T is a plane with TO at 0, 0, 0. TH makes an angle, with G.L. whose tangent is $\frac{3}{4}$; TV makes an angle with G.L. whose tangent is $\frac{3}{8}$, both towards the right. A, B, C and D are four points in the plane and their horizontal projections are respectively as follows: $a = 2, 0, \frac{5}{8}$; $b = 1, 0, -1$; $c = -\frac{1}{2}, 0, -\frac{1}{2}$, and $d = 1\frac{1}{2}, 0, 0$. Find the vertical projections of the points by using a vertical of the plane in each case.

41. Given the plane T, with TO = 0, 0, 0. TH has 2 at $2, 0, 1\frac{3}{4}$; TV has 1 at $2, 1\frac{1}{2}, 0$; apoint O, which is in the plane, has $o = 1\frac{1}{4}, 0, \frac{1}{2}$. Draw any two lines through the point O and lying in the plane T.

42. Given the three points, $A = 0, \frac{1}{4}, \frac{7}{16}$; $B = 2\frac{1}{4}, 0, 2$, and $C = \frac{1}{2}, 1, \frac{5}{16}$; to pass a plane through them.

43. Given two parallel lines AB and CD; $A = 0, \frac{3}{8}, \frac{1}{2}$; $B = \frac{5}{8}, \frac{7}{8}, \frac{1}{4}$; $C = \frac{1}{4}, \frac{3}{8}, 1$. Pass a plane through the two lines.

NOTE.—Parallel lines have parallel projections.

44. The line AB has $A = 0, \frac{1}{4}, \frac{7}{8}$; $B = \frac{3}{4}, \frac{1}{2}, \frac{1}{4}$. Pass a plane through the line AB and a point $C = 1\frac{3}{4}, 1, -\frac{3}{4}$.

45. The line AB has $A = 0, 1\frac{1}{2}, \frac{1}{4}$; $B = 1\frac{3}{8}, -\frac{1}{2}, 1\frac{1}{8}$. Pass a plane through the line AB and a point $C = 1\frac{3}{4}, 1\frac{1}{2}, -\frac{1}{2}$.

46. Given the vertices A, B and C of a triangle: $A = 0, \frac{3}{4}, 1\frac{1}{4}$; $B = \frac{3}{4}, 1\frac{1}{2}, \frac{3}{4}$; $C = 1\frac{3}{4}, \frac{1}{2}, 1$. Find the lengths of the edges of the triangle by revolving it into H about the H trace of its plane.

47. Find the traces of the plane of the three following points: $A = 0, 1\frac{1}{8}, \frac{7}{8}$; $B = 1\frac{7}{8}, 1\frac{5}{8}, \frac{1}{4}$; $C = 2\frac{1}{2}, \frac{3}{4}, -\frac{1}{4}$.

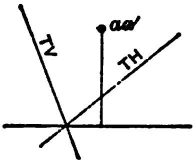
48. The H projection of a line AB has $a = 0, 0, -\frac{3}{8}$; $b = 2\frac{1}{2}, 0, 1\frac{1}{4}$. It lies in a plane T, with $TO = 2\frac{3}{4}, 0, 0$. TV has $1 = 4\frac{3}{8}, 1\frac{7}{8}, 0$. TH has $2 = 0, 0, \frac{5}{8}$. Find the other projection of the line AB.

49. $A = 0, -1, -1\frac{11}{16}$. A plane T containing the point has $TO = 1\frac{5}{8}, 0, 0$. TH has 2 at $3\frac{3}{4}, 0, 1\frac{1}{2}$. Find TV.

Analysis:—Since the point lies in the plane, a line of the plane may be drawn to pass through it, and by means of its traces, the required trace of the plane can be found.

Construction:—Draw any line through the point A, and lying in the plane T, preferably a vertical of the plane. Its H projection will be parallel to the G.L. Where it cuts TH prolonged is the H projection of its H trace; the V projection of its H trace is on the G.L. at the foot of a perpendicular through the H projection. The vertical projection, of the vertical of the plane, goes through the vertical projection of the H trace and the point a'. The V trace of T is parallel to this line and goes through TO.

50. AB and CD are two intersecting lines. $A = 0, -\frac{7}{8}, \frac{3}{8}$ and B



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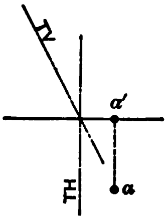

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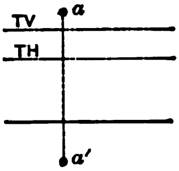

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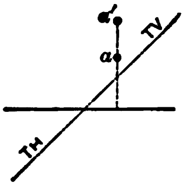


54.

= 2, $2\frac{5}{8}$, $1\frac{1}{2}$; C = $\frac{3}{4}$, $2\frac{3}{4}$, $1\frac{5}{8}$, and D* = $3\frac{3}{4}$, —, 0. A point O lying in the plane of AB and CD, has *o'* = $1\frac{3}{4}$, $\frac{7}{8}$, 0. Find *o* without using the traces of the plane of the lines.

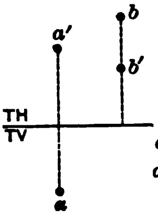
To Follow Section 51, Page 43.

51. Draw a line through the point A and parallel to the plane T.

52. Draw a line through the point A and parallel to the plane T.

53. Draw a line through the point A and parallel to the plane T.

54. Draw a line through the point A and parallel to the plane T.



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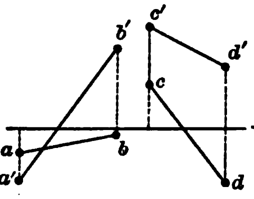

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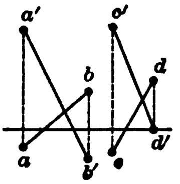

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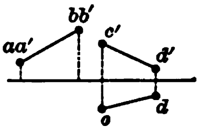

58.

55. A is a point in the plane T. Draw a line through the point B and parallel to the plane T, which is not a horizontal line.

To Follow Section 52, Page 44.

56. Pass a plane through the line AB and parallel to the line CD.

57. Pass a plane through the line AB and parallel to the line CD.

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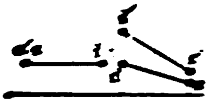
* When a coordinate is left with a —, it means that it is to be found. It is impracticable to give by coordinates, four points accurately on two intersecting lines.



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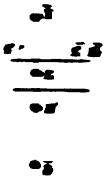
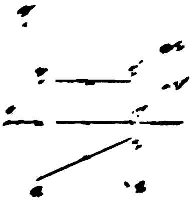


60.

58. Pass a plane through the line AB and parallel to the line CD.

59. Pass a plane through the line CD and parallel to the line AB.

60. Pass a plane through the line AB and parallel to the line CD.



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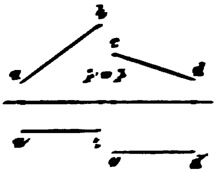

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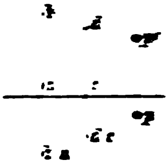

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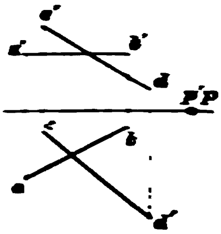


64.

TO FOLLOW SECTION 54, PAGE 46.

61. Pass a plane through the point O and parallel to the lines AB and CD.

62. Pass a plane through the point P and parallel to the lines AB and CD.

63. Pass a plane through the point P and parallel to the lines AB and CD.

64. Pass a plane through the point P and parallel to the two lines AB and CD.

TO FOLLOW SECTION 56, PAGE 48.

65. T and S are two planes with TV and TS at 45° to the G.L.

towards the right; TV intersects the G.L. at 0, 0, 0; SV intersects the G.L. at 1, 0, 0. The H traces of both intersect at 3¼, 0, ¾. Find their line of intersection.

66. T and S are two planes. T is perpendicular to V; TV making 45° with the G.L. toward the left, cutting same at 3¼, 0, 0. S cuts the G.L. at 2, 0, 0; SV makes an angle of 60° with the G.L. towards the left, and SH, 30° with the G.L. in front of, and to the right. Find the line of intersection of the two planes.

67. T and S are two planes. TV is perpendicular to the G.L., cutting it at 0, 0, 0; TH makes an angle of 30° with the G.L. in front of and toward the right. SH is perpendicular to the G.L., cutting it at 3, 0, 0; SV makes an angle of 45° with the G.L., above and toward the left. Find the line of intersection of the two planes.

68. T and S are two planes. TH and SH are perpendicular to the G.L. and cut the latter, respectively, at ¾, 0, 0 and 2¼, 0, 0. Their V traces intersect each other at a point 0, 2¼, 0. Find the line of intersection of the planes.

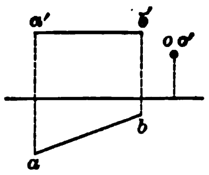
To Follow Section 57, Page 51.

69. T is a plane intersecting the G.L. at 0, 0, 0. TV makes an angle at 30° with the G.L. and TH, 45° with the G.L., both toward the right. A line has both projections parallel to the G.L. and goes through the point A = 0, 1, 1⅛. Find where the line pierces the plane.

70. AB is a line, with A = 0, − 1¼, − ¾, and B = 1¼, 1¼, ⅝. Find where it pierces a plane T with TV perpendicular to the G.L. and cutting the latter at 2⅝, 0, 0, and TH making an angle of 30° with the G.L. toward the left.

71. A line AB has A = 0, 1⅝, ¼, and B = 0, ⅝, 1. A plane T has TV and TH coinciding and at 30° to the G.L. above and toward the left. Find where the line AB pierces the plane T.

72. A plane T has both traces parallel to the G L. TV 1½ inch



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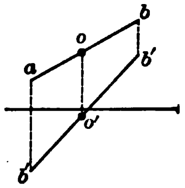

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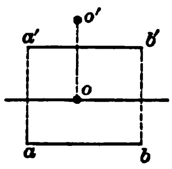

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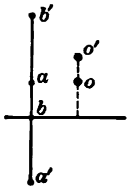


76.

above it, and TH, $1\frac{1}{8}$ inch in front of it. A line has A = 0, $\frac{1}{4}$, — $\frac{1}{4}$, and B = $1\frac{1}{8}$, $1\frac{1}{8}$, — $1\frac{1}{8}$. Find where the line AB pierces the plane T.

TO FOLLOW SECTION 59, PAGE 53.

73. Through the point O, pass a plane perpendicular to the line AB.

74. Through the point O, pass a plane perpendicular to the line AB.

75. Through the point O, pass a plane perpendicular to the line AB.

76. Through the point O, pass a plane perpendicular to the line AB.

TO FOLLOW SECTION 60, PAGE 55.

77. Plane T is perpendicular to H and V at 0, 0, 0. A point O is at $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{4}$. Find the distance from the point O to the plane T.

78. Plane T is perpendicular to H, makes an angle of 20° with V, cuts G.L. at 0, 0, 0. A point O is at $\frac{3}{4}$, $\frac{3}{4}$, $\frac{3}{4}$. Find its distance from the plane T.

79. T has its V trace parallel to the G.L., $\frac{5}{8}$ inch above it, its H trace is parallel to the G.L., $\frac{1}{4}$ inch in front of it. O is a point at 0, — $\frac{5}{8}$, — $\frac{3}{8}$. Find its distance from the plane T.

80. A plane T is perpendicular to V and makes 20° with H., cutting the G.L. at 0, 0, 0. A point O is at $\frac{3}{4}$, 0, $\frac{3}{4}$. Find its distance from the plane T.

81. A plane T has its V trace parallel to the G.L., $\frac{3}{8}$ inch below H, its H trace parallel to the G.L., $\frac{5}{8}$ inch back of V. O is any point on the G.L. Find its distance from the plane T.

82. A plane T has both traces parallel to the G.L. and coinciding; and $\frac{1}{2}$ inch above it. A point O is at $0, -\frac{3}{8}, \frac{3}{8}$. Find its distance from the plane T.

To Follow Section 63, Page 58.

83. $A = 0, 0, \frac{7}{8}$; $B = 1\frac{1}{4}, 1\frac{1}{8}, 1\frac{1}{4}$; $C = \frac{7}{8}, \frac{5}{16}, \frac{3}{8}$; $D = \frac{3}{16}, \frac{5}{8}, -$. Find the angle between the lines AB and CD.

84. A plane T has V trace at 45° with the G.L. above and towards the right, and H trace at 30° with the G.L. below and towards the left. Find the angle between the two traces.

85. A, B and C are the vertices of a triangle. $A = 0, \frac{3}{4}, 1\frac{1}{4}$; $B = \frac{3}{4}, 1\frac{1}{2}, \frac{3}{4}$; $C = 1\frac{3}{4}, \frac{1}{2}, 1$. Find its true shape and the angles between the sides.

86. $A = 0, -\frac{5}{8}, -\frac{1}{4}$; $B = 1, -\frac{3}{8}, -\frac{1}{4}$; $C = \frac{1}{2}, -1, -\frac{3}{4}$. Find the angle between the lines AB and BC.

87. $A = 0, -\frac{5}{8}, -\frac{3}{4}$; $B = 0, -\frac{5}{8}, 0$; $C = \frac{1}{2}, -1, -\frac{5}{8}$. Find the angle between the lines AB and BC.

88. $A = 0, 1\frac{3}{8}, -1$; $B = 0, \frac{3}{8}, 0$; $C = 0, -\frac{1}{2}, -1$. Find the angle between the lines A B and B C.

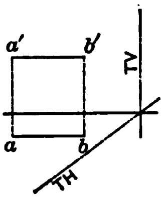
89. Given a plane T whose traces coincide and make an angle of 45° with the G.L. Choose any point on the plane, not on either trace, and draw two lines lying in the plane and making an angle of 60° with each other.

90. Given a plane T with $TO = 0, 0, 0$. A point 1 on the V trace $= 2\frac{1}{4}, -\frac{3}{4}, 0$, and a point 2 on the H trace $= 2\frac{1}{4}, 0, 1\frac{3}{8}$. A line AB in it has $A = 1, 1\frac{1}{4}, 0$, and $B = 2, 0, -$. Draw a line in the plane making an angle of 90° with AB.

91. A line AB has $A = 0, -\frac{3}{4}, \frac{3}{4}$; $B = 2\frac{3}{4}, \frac{7}{8}, -\frac{1}{8}$. Pass a plane through the line and parallel to the G.L. At a point O, which divides the line AB in the ratio of 5 to 4, draw a line making 90° with it and lying in the plane passed through it.

92. O = 5, — $\frac{3}{4}$, — $\frac{1}{2}$. It is a point in a plane T, which is perpendicular to H and makes an angle of 30° with V. Draw two lines through O making an angle of 60° with each other.

93. O = 0, — $\frac{5}{8}$, — $\frac{1}{2}$. It is a point in a plane containing the G.L. draw two lines through O and in the plane making an angle of 120° with each other.



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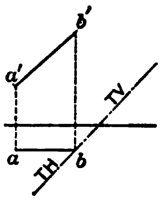

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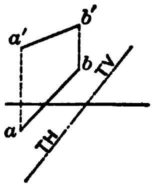

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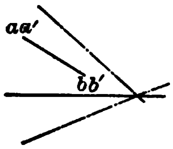


97.

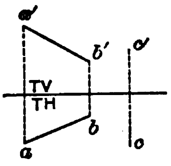
To Follow Section 66, Page 59.

94. Project the line AB on the plane T.

95. Project the line AB on the plane T.

96. Project the line AB on the plane T.

97. Project the line AB on the plane shown.



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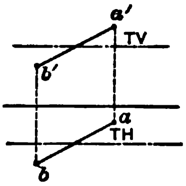

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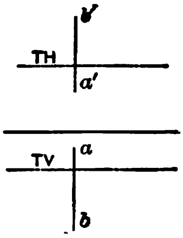

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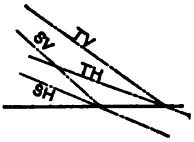
98. Project the line AB on the plane T determined by the point C.

99. Project the line AB on the plane T.

100. Project the line AB on the plane T.



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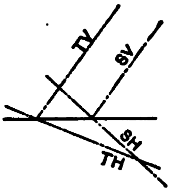

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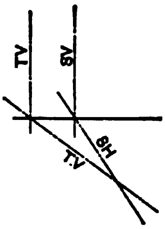

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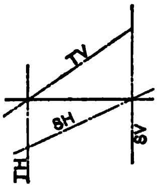


104.

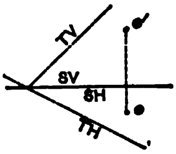
To Follow Section 68, Page 62.

101. Find the angle between the planes T and S.

102. Find the angle between the planes T and S.

103. Find the angle between the planes T and S.

104. Find the angle between the planes T and S.



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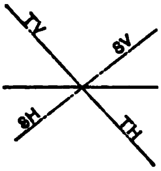

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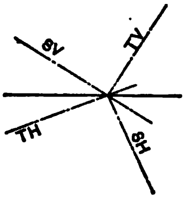

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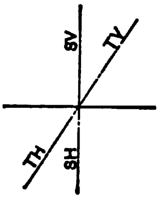


108.

105. Find the angle between the planes T and S, the latter being determined by the point O.

106. Find the angle between the planes T and S.

107. Find the angle between the planes T and S.

108. Find the angle between the planes T and S.

To Follow Section 77, Page 75.

109. A plane T has both traces coinciding and making an angle of 45° with the G.L. toward the right. Draw the projections of a triangular pyramid of $2\frac{1}{2}$ inches altitude, and equilateral triangular base one inch on a side, an edge of the base in H and plane of base in T.

110. A plane T has TV at 60° to the G.L. above and to the right. TH at 45° to the G.L., below and toward the right. A base of a cube lies in it with one corner in V and a diagonal of the base parallel to V. The cube is $1\frac{1}{2}$ inches on edge. Draw its projections.

111. Find the locus of points equi-distant from the two points: $A = 0, 1, \frac{3}{4}$; and $B = 1\frac{3}{4}, 1\frac{1}{2}, 1\frac{1}{4}$.

112. Find the locus of points in space equi-distant from the two following planes, T and S. Point 1 of T[*]$= 0, \frac{1}{2}, 0$, and $3 = 2\frac{1}{2}, \frac{1}{2}, 0$. Point $2 = 0, 0, 1$, and $4 = 2\frac{1}{2}, 0, 1$. Point 1 of $S = 0, 1\frac{3}{4}, 0$, and $3 = 2\frac{1}{4}, 1\frac{3}{4}, 0$. Point $2 = 0, 0, 1$, and $4 = 2\frac{1}{2}, 0, 1$.

113. Find the locus of points in space equi-distant from the points: $A = 0, 1, \frac{3}{4}$, and $B = \frac{7}{8}, \frac{3}{4}, \frac{7}{16}$, and $C = 1\frac{9}{16}, 1\frac{3}{4}, 2$.

To Follow Section 83, Page 82.

114. $A = 0, 1\frac{3}{16}, \frac{1}{2}$; $B = \frac{7}{8}, 0, 0,$; $C = 1\frac{5}{8}, 1\frac{3}{4}, \frac{1}{8}$. Find the locus of points in space equi-distant from the lines AB and BC.

115. $N = 0, -\frac{1}{4}, -1\frac{3}{8}$; $P = \frac{5}{8}, -1\frac{1}{4}, -1$; $O = 1\frac{3}{8}, -\frac{1}{2}, -\frac{9}{16}$. Draw the locus of points in space equi-distant from the lines NO and OP.

To Follow Section 102, Page 101.

116. MN is the axis of an oblique cylinder, with circular base $1\frac{1}{2}$ inches in diameter, in H. $M = 0, 0, \frac{7}{8}$; $N = 2, 1\frac{3}{4}, 1\frac{1}{4}$. The bases are parallel; O, P and Q are points on the surface; $o' = \frac{1}{2}, \frac{3}{4}, 0$; $p' = 1, 1\frac{3}{8}, 0$; $q = 2\frac{1}{8}, 0, \frac{5}{8}$; find their corresponding projections; draw a tangent plane to the cylinder at Q.

117. MN is the axis of an oblique cylinder, having a circular base in H, $1\frac{1}{2}$ inches in diameter. The bases are parallel; $M = 0,$

---

[*] The odd numbers are points on the V trace, the even those on the H trace.

0, 1; N = $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{8}$. P is a point on the cylinder at $\frac{1}{2}$, $1\frac{1}{8}$, — ; draw a plane tangent to the cylinder at P; draw another plane normal to the cylinder at the point P.

TO FOLLOW SECTION 114, PAGE 112.

118. Develop the cylinder of problem 117.

TO FOLLOW SECTION 117, PAGE 116.

119. A righ circular cone of $\frac{3}{4}$ inch diameter of base, $1\frac{3}{4}$ inches altitude, has its axis parallel to, and equi-distant from, V and H in the 3rd angle; center of base is at 0, $-\frac{5}{8}$, $-\frac{5}{8}$. A point P on its surface has its H projection at $\frac{9}{16}$, 0, -1. Draw a plane tangent to the cone at P.

120. An oblique cone, with circular base in H, $1\frac{1}{2}$ inches in diameter, and center on the G.L, at 0, 0, 0, has apex at $1\frac{5}{16}$, $1\frac{1}{8}$, $1\frac{5}{16}$. A point P on its surface is at $\frac{7}{16}$, —, $\frac{3}{4}$; draw a tangent plane to the cone at P.

TO FOLLOW SECTION 119, PAGE 119.

112. An oblique cone, with circular base in H, $1\frac{1}{2}$ inches in diameter, and center on the G.L. at 0, 0, 0, has apex at $1\frac{5}{16}$, $1\frac{1}{8}$, $1\frac{5}{16}$. A line MN has M = $1\frac{1}{2}$, $\frac{1}{16}$, $-\frac{1}{16}$; N = $2\frac{1}{8}$, 1, -1; draw a tangent plane to the cone and parallel to the line MN.

122. A right circular cone, of $\frac{3}{4}$ inch diameter of base $1\frac{3}{4}$ inches altitude, has its axis parallel to, and equi-distant from, V and H, in the 3rd angle; center of base is at 0, $-\frac{5}{8}$, $-\frac{5}{8}$. A line BC has B = 2, $-\frac{3}{4}$, $-\frac{7}{16}$; C = $2\frac{7}{8}$, $-\frac{1}{4}$, -1. Draw a plane tangent to the cone and parallel to the line BC.

123. A right circular cone has base in H, 2 inches in diameter, $2\frac{1}{2}$ inches altitude, and center at $2\frac{1}{2}$, 0, $1\frac{1}{2}$. MN is a line with M = 0, $\frac{1}{4}$, $1\frac{1}{4}$, and N = $1\frac{1}{4}$, $1\frac{1}{4}$, $\frac{1}{4}$. Pass a plane tangent to the cone and parallel to the line MN.

124. Discuss the conditions of problem 123 when (1), the given line has the same inclination to the plane of the base of the cone that the elements of the cone do; (2), when the given line makes a less angle, with the plane of the base of the cone, than the elements do.

To Follow Section 123, Page 122.

125. An oblique cone has apex at 0, 1⅝, 2. The base touches H at the point 1⅝, 0, 1⅝, is a circle in H projection, and its plane is perpendicular to V and makes an angle of 30° with H. A plane T, perpendicular to the axis of the cone, cuts the G.L. at ⅝, 0, 0. Find the curve of intersection of the cone and the plane.

To Follow Section 126, Page 125.

126. Develop the cone of problem 125.

To Follow Section 150, Page 161.

127. Draw the contour of an oblique helicoid of uniform pitch, with the following conditions: One helical directrix, 1½ inches in diameter, second helical directrix, 4 inches in diameter; pitch of helices 4 inches; angle of elements with the axis 15°. Take the H plane perpendicular, and the V plane parallel to the axis of the helical directrices. Plot the curves of at least three points of the generatrix, in addition to those in which it touches the two given directrices.

128. Draw the contour of a right helicoid of uniform pitch, with the following conditions: One helical directrix 1½ inches in diameter, second helical directrix 4 inches in diameter, pitch of helices 4 inches. Take the H plane parallel to the plane directer, and the V plane parallel to the axis of the helical directrices. Plot the curves of at least three points of the generatrix, in addition to those in which it touches the two given directrices.

To Follow Section 159, Page 168.

129. An oblate spheroid rests on H and touches V. The minor axis is vertical and is 2 inches long, touching H at 3, 0, 1¼; the major axis is 2½ inches long. P is a point on the upper surface, and its V projection is at 3¾, 1¼, 0. Pass a plane through P and the minor axis. Show the curve of intersection of the plane with the spheroid; draw a plane tangent to the surface at P.

To Follow Section 161, Page 170.

130. A sphere, 2¼ inches in diameter, rests on H, with center 1¼ inches from V in the 1st angle. A point P, on the front surface of the sphere, has its V projection ⅜ inch above the center and ⅝ inch to the right. Draw a plane tangent to the sphere at the point P.

131. An ellipsoid of revolution has a major axis 1¾ inches long, minor, 1⅛ inches. It lies in the 3rd angle, touching the H plane, the major axis is parallel to V, 1 inch from it, and is perpendicular to H. A point P, on the surface, has its V projection ⅜ inch to the left of the axis, and 1⅝ inch below H. Draw a plane tangent to the surface at the point P.

132. A circle, 1 inch in diameter, revolves about a vertical axis which touches H at a point 1⅛, 0, 1⅞, thus generating a torus. The center of the circle is at a distance of 1 inch from the axis, and the torus rests on H. A point upon the surface is at a distance of ¾ inch from the axis and 1¼ inches from V; it may have several positions, show them and pass planes tangent to the surface at each.

To Follow Section 167, Page 180.

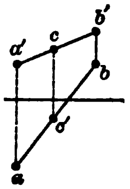
133. A line AB has A = 0, 0, 3½; B = 2⅜, 3, 1⅛; it revolves about an axis perpendicular to H, whose H projection is the point 1¼, 0, 1⅞, thus generating a hyperboloid of revolution of one nappe. Draw the upper base generated by the point B, and the lower, generated by the point A; draw the circle of the gorge, and the principal meridian plane; draw through B an element of the second generation.

Locate a point P, on the surface, at 1¾, —, 1¾; a point R at ⅛, ⅞, —. Draw a plane tangent to the surface at P.

Find the curve of intersection, with a plane S, cutting the G.L. at 3⅜, 0, 0, and whose V and H traces make angles of 30° with the G.L. toward the right, above and below the G.L., respectively.

To Follow Section 174, Page 187.

134. Two oblique cones, with circular bases in H, intersect each other. One cone has base 2⅛ inches in diameter, its center being at 0, 0, 2, and apex at 2⅛, 3¼, ¾; the other cone has base 3 inches in diameter, its center being at 2⅞, 0, 2⅜, and apex at ⅜, 2¼, ⅛. Draw the curve or curves of intersection of the two cones, using a sufficient number of points to obtain a good curve.



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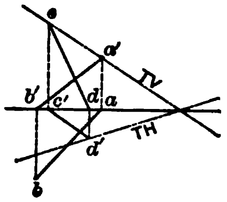


136.

135. Prove that the point C does not lie on the line AB.

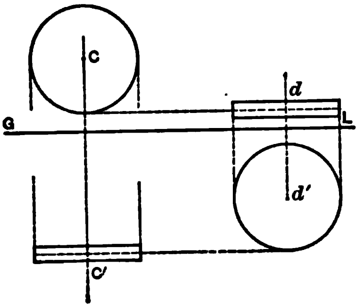
136. Prove that the two lines AB and CD do not lie in the plane T.

137. A = 0, ⅜, ⅜; B = ⅞, 1⅛, ⅞; P = 1⅜, ¾, ⅞. A plane T has TO = 1¾, 0. 0; TV makes an angle of 45° with the G.L. above and to the right; TH makes an angle of 45° with the G.L. below and to the right. Draw, through P, a plane parallel to the line AB, and perpendicular to the plane T.

138. P = 0, ¾, ¾; A = ¾, ¼, 1; B = 2⅜, ⅞, ⅛. Draw a plane through the line AB and distant 1 inch from P.

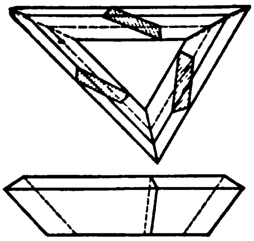
139. A plane T has TO = 0, 0, 0; TV makes an angle of 30° with the G.L. above and to the right; TH makes an angle of 45° with with the G.L. below and toward the right. A point A, in the plane, is at 1, ⅜, —. Through A draw a line making an angle of 30° with H.

140. A line AB makes an angle of 30° with H; its V trace, A, is at 0, ⅞, 0; the end B has its H projection at 1⅝, 0, 1. Draw a line through the H trace of AB, lying in H, and making an angle of 45° with AB.



141.

141. Given the two pulleys as shown, and in 3rd angle projection, and, assigning dimensional values to them, etc., find the position of the center, and the direction of the axis of an idler, of any given diameter, which will take the belt from the left hand pulley and deliver it to the right hand one without its slipping off.



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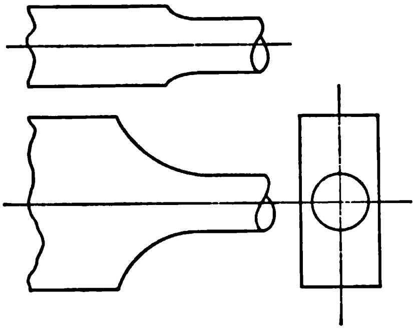


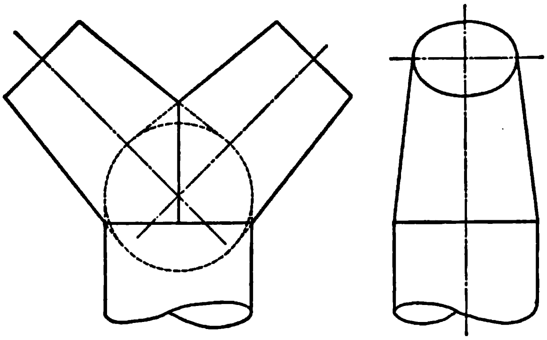
143.

142. Assigning numerical values to the edges of the form shown, find:

(1). The angles between the sides of the solid.

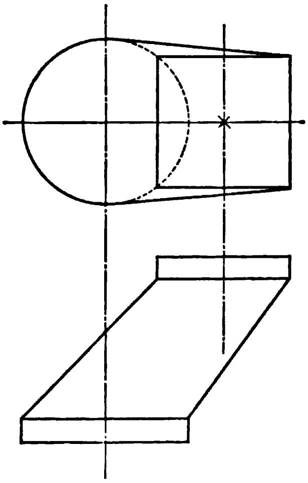
(2). The angles of the bevel of the edges between the different sides.

143. Find the curve of intersection between the stem and end of the connecting rod as shown.



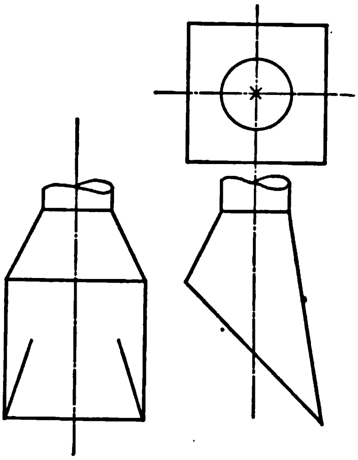
144.

144. Given the 'two way' flue, as shown, and assigning dimensional values to the form, develop each piece as for sheet metal work.



145.

145. Given the form shown, which is a 'cylindrical to square' offset, develop it by triangulation, as discussed in the appendix, namely, take four triangles, whose bases are the edges, respectively, of the square opening, and whose apexes are in the cylindrical portion below. Between these, divide the cylindrical opening into a number of parts, the bases of another set of triangles whose apexes are in the corners of the square opening above.



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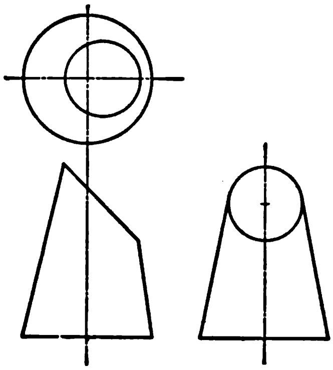
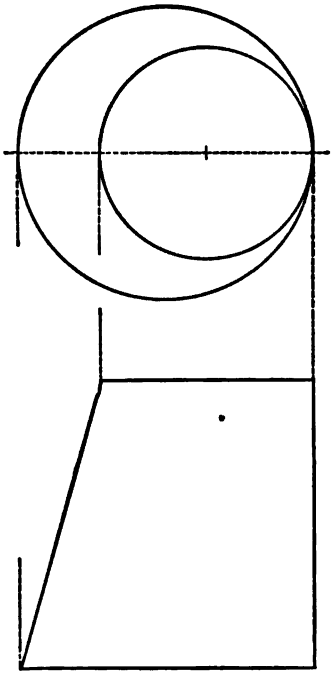


147.

146. Given the form shown, which is a cylindrical chimney above a square opening, and, assigning to it dimensional values, develop the surface, as for sheet metal work, by the method of triangulation described in the appendix, namely, take first, four triangles, whose bases are, respectively, in contact with the four lower edges, and apexes in the neck above; then a series of triangles whose bases are contiguous and form part of the neck, with sides touching and apexes at the corners where the four lower edges meet.

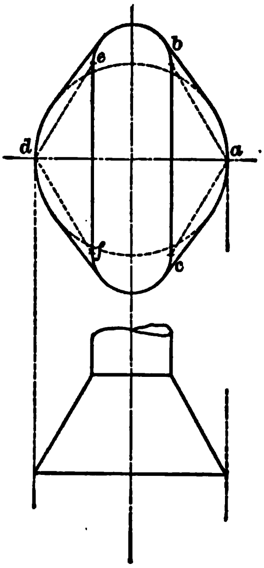
147. Develop the frustum of the conical figure shown, by the method of triangulation discussed in the appendix, namely, divide the surface into a number of triangles, whose bases are oppositely directed, and lie in the upper and lower bases of the form, respectively, and whose sides are touching one another.



148.

148. Given the form shown, which is analogous to the slope sheet of a locomotive boiler, and assigning dimensional values to it, develop it, as for sheet metal work, by the method of triangulation, discussed in the appendix, namely, divide the upper and lower bases into a large number of equal parts; let the spaces between the divisions be the bases of a set of triangles, one set in the upper curve, the other set in the lower curve, and the apexes of each set in the opposite curve, the edges of the triangles coincident.

149. Given the form shown, which is a 'round to flat' flue, and assigning dimensional values to it, develop it as for sheet metal work, by the method of triangulation as described in the appendix namely, take two triangles, with apexes at *a* and *d*, respectively



149.

and with bases *bc* and *ef* respectively; then, divide up the remainder of each base into a large number of parts, each one into the same number. Regard the divisions in each base as the bases of a set of triangles with apexes in the opposite base, at the points of division on that base.

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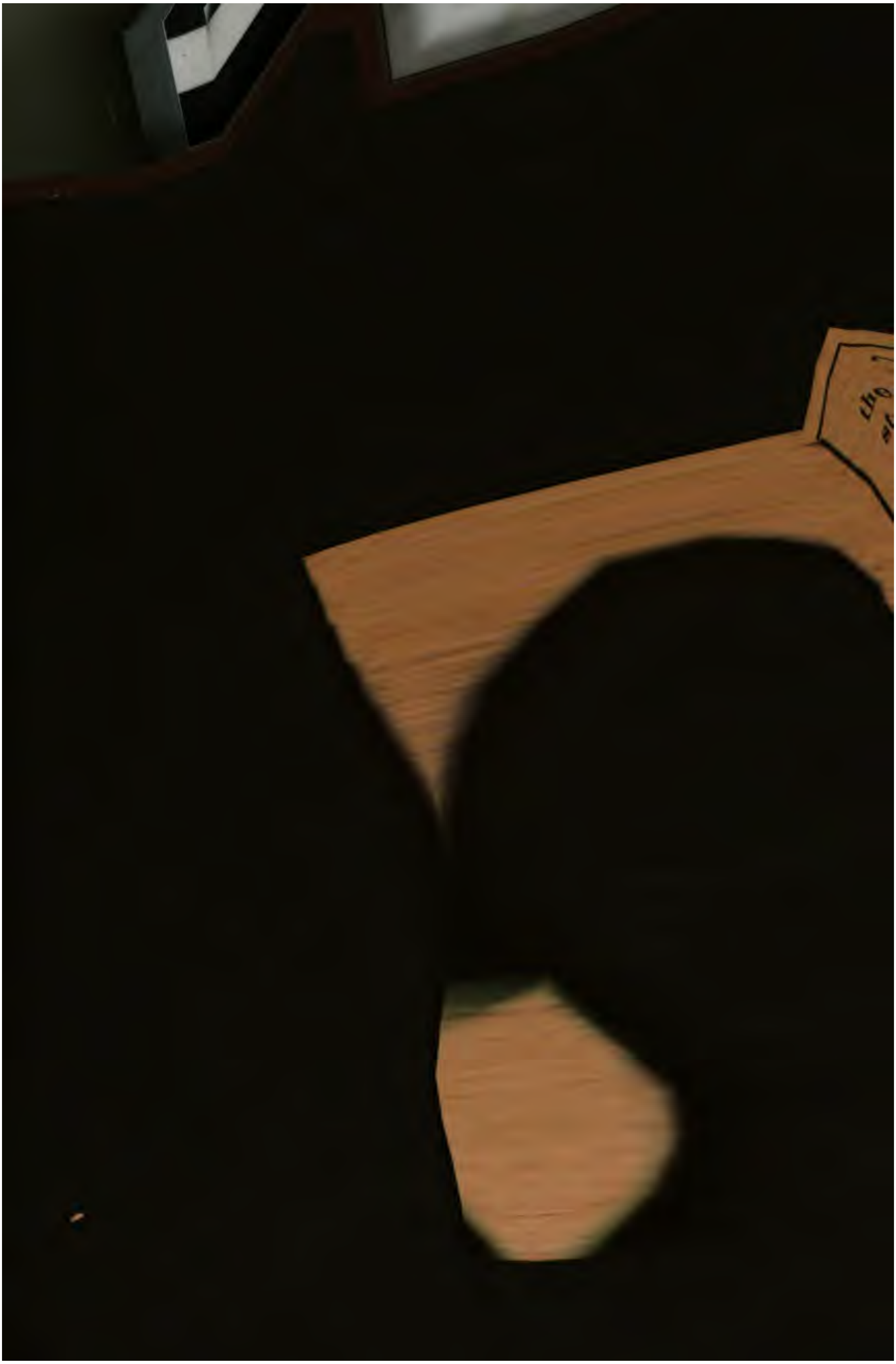
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